Poverty Trap and Endogenous Population*

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Abstract

In this paper, we develop a growth model in which human being is a production factor which can be combined with a fixed factor, say land, to produce a homogeneous commodity. Saving, so to speak, can only be made through having children, the number of which is an endogenous decision to the household. In this context, we show that the economy may run into a poverty trap with a subsistence level per capita consumption. However, we also demonstrate that the economy can escape from this unappealing long run situation through a suitable technological shift or an appropriate child-rearing tax. For such an escape, the technological shift must be non-neutral in the sense that it modifies the ratio of factor’s marginal productivity.

Keywords: Poverty trap, Endogenous population, Endogenous growth

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1. Introduction

Recent resurrected interests in the theoretical population problem point to the important connection between economic growth and the fertility decision in the new household economics. In this vein, the pessimistic vision set forth two centuries ago in Malthus’s "Essays in the Principle of Population" comes back to haunt the debate on the thorny subject of economic sustainability. The human carrying capacity of the earth at the present state of technology has recently being well documented in Cohen (1995) where, once again, the question of which population size that would keep per capita consumption sustainable at some decent level other than the mere subsistence is raised. Does population growth would ultimately equilibrate the per capita consumption to the level of subsistence? In the affirmative, are there some means to escape from this rather desperate perspective? This paper addresses to these questions.

Many theorists - some of them the most prominent - in the neo-classical tradition would have just given to this vision little, if any, importance. Population growth remains unbounded in Barro and Becker (1989) where fertility decisions are endogenous, but labor can be substituted by physical capital that can be accumulated without limit. The same conclusion is reached when, instead, the human capital plays the role of factor substitution in Becker, Murphy and Tamura (1992). However, if capital is to be understood in a larger sense and embodies the environmental category, its accumulation would ultimately come to an end, then unlimited factor substitution is no longer possible. What should then happen? Is it possible, as pointed out by Kremer (1993), that technological progress induces just population growth thus leaving the economy emprisoned in a poverty trap? One very interesting attempt to address this question is given by Galor and Weil (1998). They build a model in which, in a first regime, the economy has a globally stable steady state in which the per capita consumption is equal to the subsistence level. There, technological progress is slow and completely wiped out by population growth. In a second regime, the growth rates of technology increases. Population growth absorbs much of the output growth but per capita consumption rises slowly. The economy undergoes a demographic transition in which there is a reversion of the positive relationship between income per capita and population growth described by Kremer (1993). In a third and last regime, population growth is moderate or even negative, and the per capita consumption rises rapidly. Two forces drive the transition between regimes. First, technological progress is driven both by increases in the size of the population and by
increases in the average level of education. Second, technological progress creates a state of disequilibrium, which raises the return to human capital and induces parents to substitute child quality for quantity. Nevertheless, as pointed out by the authors, the model is not fully applicable to countries that are developing today.

The objective of this paper is to give another description of the escape from a poverty trap for a developing economy. We consider in the sequel human being as both the sole asset and production factor which can be combined with a fixed factor, say land to produce a homogeneous commodity. Saving, so to speak, can only be made through having children, the number of which is an endogenous decision to the household. The utility of the representative household takes the dynastic form so that the decentralized-decision making of all agents of different generations can be subsumed to the planning Ramsey problem. We demonstrate that, in this framework, a persistent poverty trap may prevail. However, we also show that the economy can escape from this unappealing long run situation through a suitable technological progress or an appropriate child-rearing tax. As for the technological progress, we show that the neutral progress, whether endogenous or not, would never detract the long run consumption from the subsistence level which characterizes the poverty trap.

The model in this paper is put forth in the next Section. In Section 3, the optimal solution to our dynamic planning problem is carried out in some details, with particular attention given to the poverty trap. In Section 4, we show that the economy can escape from a poverty trap through a non-neutral technological progress or an appropriate child-rearing tax. In the final section, we sum up our main results and offer some generalization whenever possible.

2. The Model

The model considered in this paper is a variant of the Solow-Swan model of economic growth where, instead of physical asset, "people" constitutes the only form of productive "capital". In discrete time, each individual agent living for 1 period earns an income $y_t$ has to make decision about his consumption $c_t$, the number of his offsprings, $n_t$, who become themselves adults at $t+1$. Suppose that the utility of each agent is a function of its own consumption $U(c_t)$ and the utility of all of his descendants:

$$V_t = U(c_t) + n_t\delta\alpha(n_t)V_{t+1},$$
where $\delta$ denotes the discount factor of time preference, and where $\alpha(n)$ measures the degree of altruism of the parent toward each child. This degree $\alpha$ is likely a decreasing function of $n_t$, but following Becker and Barro (1988), $n_t \alpha(n_t)$ is assumed to be strictly increasing and concave in terms of $n_t$. For our purpose, let $\alpha(n_t) = n_t^{-\epsilon}$ with $0 < \epsilon < 1$, that is the degree of altruism is of constant elasticity with respect to the number of children. At time $t = 0$, assume that there are $N_0$ identical individuals, hereafter called the patriarchs, each of them gives birth to $n_0$ children. If $N_1$ is the population at $t = 1$, then $N_1 = N_0 n_0$. Similarly, $N_2 = N_1 n_1 = N_0 n_0 n_1$, etc., and $N_t = N_0 \prod_{s=0}^{t-1} n_s$. Because the utility level of the agent’s children is itself a function of the utility level his own descendants, we can, by substitution, write the patriarchs’ utility function as

$$V_0 = \sum_{t=0}^{\infty} \delta^t N_t \alpha(n_t) U(c_t),$$

where $N_t$ is the population at time $t$. The analogous of the patriarchs’ utility function in continuous time version (see Barro and Becker (1989)) is

$$V_0 = \int_{0}^{\infty} e^{-\rho t} N_t \alpha(n_t) U(c_t) dt,$$

$0 < \rho < 1$, $\rho$ being the rate of time preference taken to be a constant parameter, and $N_t = N_0 e^{\int_{0}^{t} (n_s - 1) ds}$.

In view of focusing on the issue of endogenous population, we suppose a simple production process which requires essentially the use of labor, the supply of which depends on agent’s fertility decisions. The aggregate production in the economy can be written as $F(L, N_t)$ where $L$ represents a factor available in fixed quantity, and $N_t$, the population taken to be the labor in use. It is natural to interpret $L$ as the total amount of land of a given level of soil fertility.

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1Here the rate of utility discount is composed of the rate of time preference adjusted by the family size and the degree of the patriarch altruism. In continuous time, if the utility discount factor is denoted by $\theta(N_t, t)$, then $\theta / \theta = -[\rho + \epsilon (n_t - 1)]$. The patriarchs’ utility, $\int_{0}^{\infty} N_t \theta(N_t, t) U(c_t) dt$, can be written as $\int_{0}^{\infty} N_t e^{-\rho t} e^{-\epsilon \int_{0}^{t} (n_s - 1) ds} U(c_t) dt$. Using the definition of $N_t$, this utility can be easily seen as (1). Note that $n_t$ is endogenous, therefore our formulation is reminiscent of Epstein(1987) which addressed a completely different question in dynamics.

2This is a short-hand representation of the market allocation: on the factors supply side, assume that each individual is entitled to have an equal right on land, i.e. $\frac{L}{N_t}$, and owns one unit of labor. On factors demand side, assume that the production in the economy is conferred to a profit maximizing firm which pays labor and rents land at the marginal productivities $w = F_N$ and $r = F_L$. Factors market clearing conditions and a production under constant
At each time, production is allocated between consumption and the cost of having $n_t$ children which includes parent time, education, food, etc. If we consider the marginal cost of having one children, $\beta$, as a constant, we have the following resources constraint at time $t$:

$$ F (T, N_t) = N_t c_t + \beta N_t n_t. \quad (2) $$

In this framework, since $\dot{N}_t = (n_t - 1) N_t$, equation (2) can be cast in terms of the following diffusion process:

$$ N_t = \frac{1}{\beta} F (T, N_t) - \frac{1}{\beta} N_t c_t - N_t, \quad N_0 \text{ given.} \quad (3) $$

The problem of individual living at time $t$ is to choose his consumption $c_t$ and his number of offspring $n_t$ under the budget constraint (see footnote 2). Since his utility depends not only on his own consumption but also on the well being of all of his descendants, this recurrent relationship allows us to subsume utilities of immediate as well as all ensuing descendants in a single patriarch’s utility function (1). Note that this function satisfies the Strotz-consistency requirement, hence all individual decisions could be studied by examining the patriarch’s decisions about the number of offspring and of consumption stream over the whole time horizon. This problem therefore consists of

$$ \max_{\{c_t\}_{t=0}^{\infty}} \int_0^\infty e^{-\mu t} N_t^{1-\epsilon} U(c_t) dt $$

subject to $\dot{N}_t = \frac{1}{\beta} F (T, N_t) - \frac{1}{\beta} N_t c_t - N_t, \quad N_0 \text{ given.}$

which is a planning problem. It may be worthwhile to put forth the following remarks:

1- "People" as the sole form of capital, being only conceivable in pre-industrialist society, constitutes the unique vehicle of saving. In continuous time, each individual lives one (however small) lap of time, thus this saving which should do more than offsetting the (instantaneous) depreciation of capital in order to returns to scale imply $y = w + r \frac{F_t}{N_t}$, where $y$ denotes the individual income. The budget constraint for each individual is therefore $\frac{F_t}{N_t} = c_t + \beta n_t$ which amounts to nothing but the aggregate constraint (2) in the text.
avoid the event of a vanishing society. With identical individuals, this means \( n_t > 0, t \in [0, \infty) \), must hold. Accordingly, the constraint (2) can be rewritten as

\[
c_t < F(T, N_t)/N_t.
\]

2- Problem (P) is a non-standard one-sector economic growth model. Here the population (state variable) is embedded in the objective function (1). Hence steady state equilibria, if they exist, may be multiple (see Kurz (1968)). In order to escape from the issue of multiplicity of equilibria, and to highlight the problem of the poverty trap, we adopt the following utility functional form:

\[
U(c_t) = \begin{cases} 
  c_t^{1-\sigma}/(1-\sigma) & \text{if } c_t \geq c_s \\
  -\infty & \text{if } c_t < c_s
\end{cases}
\]

where \( c_s \) stands for the subsistence level below which the utility loss is infinite\(^3\), and \( 1/\sigma \), a constant, stands for the elasticity of substitution. Of course, our problem should take into account the additional constraint \( c_t \geq c_s \). We now turn to the analysis of our problem under the deterministic setting.

3. Endogenous Population and the Poverty Trap

Although not quite essential, we now assume that the production is of Cobb-Douglas type to ease calculations. Thus

\[
F(T, N_t) = T^{1-\gamma}N_t^\gamma = AN_t^\gamma, 0 < \gamma < 1,
\]

so that our problem is

\[
\max_{\{c_t\}_{t=0}^\infty} \int_0^\infty e^{-\beta t}N_t^{1-\gamma}[(c_t - c_s)^{1-\sigma}/(1-\sigma)]dt
\]

subject to

\[
\dot{N}_t = \frac{1}{\beta}AN_t^\gamma - \frac{1}{\beta}N_t c_t - N_t, N_0 \text{ given},
\]

\[
c_t - c_s \geq 0; c_t \leq AN_t^{\gamma-1}.
\]

\(^3\)Note here that, although the utility loss is infinite if consumption is below the subsistence level, the marginal utility of an increase of consumption over the subsistence level

\[
\lim_{c \to c_s} u'(c) = c^{-\sigma}
\]

is finite. Obviously, if this marginal utility was infinite, the poverty trap will have been excluded because the agent will never choose a path that leads to a steady state where \( c_\infty = c_s \).
If \( \{c_t, N_t\}_{t=0}^{\infty} \) is the optimal path, then it satisfies Pontryagin conditions:

\[
N_t^{1-\sigma}c_t^{-\sigma} - \frac{\pi_t N_t}{\beta} + \lambda_t - \varphi_t = 0, \tag{5}
\]

\[
\dot{\pi}_t = \rho \pi_t - \frac{(1 - \epsilon)}{(1 - \sigma)} N_t^{-\sigma}c_t^{1-\sigma} - \frac{\gamma}{\beta} \left( AN_t^{\gamma-1} - \frac{1}{\beta} c_t - 1 \right) + \varphi_t \left( AN_t^{\gamma-1} - c_t \right), \tag{6}
\]

\[
\dot{N}_t = \frac{1}{\beta} AN_t^{\gamma} - \frac{1}{\beta} N_t c_t - N_t, \tag{7}
\]

\[
\lambda_t \left[ c_t - c_s \right] = 0; \quad \lambda_t \geq 0; \quad c_t - c_s \geq 0, \tag{8}
\]

\[
\varphi_t \left[ AN_t^{\gamma-1} - c_t \right] = 0; \quad \varphi_t \geq 0; \quad AN_t^{\gamma-1} - c_t \geq 0. \tag{9}
\]

where \( \pi_t \) represents the efficient price of capital (one unit of population, or equivalently, a child). Equation (5) is an arbitrage condition between consumption and saving, the latter being made by the individual through having children. It stipulates that at each point in time the current marginal cost in terms of consumption must be equal to the current marginal value of a child. On the other hand, equation (6) appropriately rewritten stipulates that the net total rate of return on the investment in offspring must be equal to the rate of time preference.

Recalling remark 1 in the preceding section, \( n_t > 0 \), and thus (4) should delimit the feasible region in the plane \( (c_t, N_t) \) for the optimal path. Finally, the solution path \( \{c_t, N_t\}_{t=0}^{\infty} \) satisfying (5) to (9) is optimal provided the transversality conditions

\[
\lim_{t \to \infty} e^{-\rho t} \pi_t N_t = 0, \tag{10}
\]

\[
\lim_{t \to \infty} e^{-\rho t} \pi_t \geq 0, \tag{11}
\]

and \( 0 \leq N_t \leq \overline{N} \) for some \( \overline{N} < \infty \). \tag{12}

In order to characterize in further details this optimal path in the phase plane \( (c_t, N_t) \), let us derive \( \dot{c}_t / c_t \) from (5) and (6). Assuming that the two constraint are not binding \( (\lambda_t = 0 \text{ and } \varphi_t = 0) \) and differentiating totally (5) with respect to \( t \), then substitute into (6), we get

\[
-\sigma \frac{\dot{c}_t}{c_t} = (1 + \rho - \epsilon) - \frac{1 - \epsilon}{\beta(1 - \sigma)} c_t + \frac{1 - \epsilon}{\beta} c_t + \frac{\epsilon - \gamma}{\beta} AN_t^{\gamma-1}. \tag{13}
\]

Note that \( \dot{\pi}_t = \dot{N}_t = 0 \) would readily imply \( \dot{c}_t = 0 \), thus the steady state equilibrium could be determined by the locus \( \Phi(c, N) \mid_{N_t=0} \) and \( \Psi(c, N) \mid_{c_t=0} \). The locus \( \Phi \) is given by
\[
c_t + \beta - AN_t^{\gamma-1} = 0, \quad (14)
\]
while the locus \(\Psi\) is given by
\[
c_t - \frac{\beta(1+\rho-\epsilon)}{1-\epsilon} \frac{1-\sigma}{\sigma} - \frac{\epsilon - \gamma}{1-\epsilon} \frac{1-\sigma}{\sigma} AN_t^{\gamma-1} = 0 \quad \text{when } \lambda_t = 0 \text{ and},
\]
\[
c_t - c_s = 0 \quad \text{when } \lambda_t > 0. \quad (15)
\]
The steady state equilibrium is then determined by the intersection of \(\Phi\) and \(\Psi\Psi\) (see Figure 3.1). In order to describe in further details the dynamic of our new setting, let us define \(\hat{c}\) and \(\hat{N}\) by the intersection of \(\Phi\) and \(\Psi^2\Psi\) which is given by
\[
c_t = \frac{\beta(1+\rho-\epsilon)}{1-\epsilon} \frac{1-\sigma}{\sigma} - \frac{\epsilon - \gamma}{1-\epsilon} \frac{1-\sigma}{\sigma} AN_t^{\gamma-1} = 0. \quad (16)
\]
Thus, we have
\[
\hat{c} = \frac{\beta(1-\sigma)(1+\rho-\gamma)}{\sigma(1-\epsilon) + (1-\sigma)(\gamma-\epsilon)}, \quad (17)
\]
\[
\hat{N} = T \left[ \frac{\beta(1-\sigma)(1+\rho-\gamma)}{\sigma(1-\epsilon) + (1-\sigma)(\gamma-\epsilon) + \beta} \right]^{\frac{1}{\gamma-1}}. \quad (18)
\]
The value of \(\hat{c}\) and \(\hat{N}\) will have been the steady state equilibrium values of the problem in absence on the subsistence level constraint. With the constraint, the steady state equilibrium values are
\[
e^\infty = \begin{cases} 
  c_s & \text{if } c_s > \hat{c} \\
  \hat{c} & \text{if } c_s \leq \hat{c}
\end{cases}, \quad (19)
\]
\[
N^\infty = \begin{cases} 
  T \left[ c_s + \beta \hat{N} \right]^{\frac{1}{\gamma-1}} & \text{if } c_s > \hat{c} \\
  \hat{N} & \text{if } c_s \leq \hat{c}
\end{cases}. \quad (20)
\]

We now proceed to use the familiar phase diagram to characterize the optimal path and its dynamic properties. At the outset, recalling that in positive orthant of the plane \((c, N)\) all feasible paths are contained the region delimited by (4) and \(c_t - c_s \geq 0\). As for the locus \(\Psi\), one can easily observe that it has a positive slope when \(\epsilon < \gamma\) and a negative slope when \(\epsilon > \gamma\). Since the dynamics involved in this analysis is not essentially different in the two cases, we concentrate on the former which has a positive slope. On the other hand, the locus \(\Phi\) has a negative slope. Thus, the steady state equilibrium clearly exits and is unique. Figure 3.1 shows the phase diagram analysis of the problem.
Figure 3.1: Dynamics of Population
Essentially, there are two cases. The poverty trap is obtained as an equilibrium outcome only in the second case. However, a discussion on the first case sheds light on the dynamics involved. The long run equilibrium is determined by the intersection of the loci \( \Phi(c, N) \mid_{N_t=0} \) and \( \Psi(c, N) \mid_{c_t=0} \) as point \( E \) in Figure 3.1. In this first case, the steady state per capita consumption \( \hat{c} > c^*_t \), and therefore the poverty trap could not prevail. It is familiar that the steady state equilibrium is stable in the sense of a saddle point. However the monotonicity of the convergence toward it is not independent of the initial condition as with the standard one-sector growth model. Because of the constraint (4) depicted as the accessible frontier \( \Gamma(c, N) \) which delimits the region in which \( c_t - AN_t^{\gamma-1} < 0 \), the dynamics of the system exhibits this peculiar, but interesting, characteristic. For, let \( \phi(c, N) = 0 \) denotes the trajectory \( \{c_t, N_t\} \) which describes the saddle-path converging to the steady state \( E \). This saddle path clearly has a positive slope. Since \( \Gamma(c, N) \) has a negative slope, their intersection is uniquely determined at \( \bar{N} \), where \( \bar{N} \) is implicitly defined by \( \phi \left( A\bar{N}^{\gamma-1}, \bar{N} \right) = 0 \). If \( N_0 < \bar{N} \), one can always choose the optimal initial consumption located on the saddle path which insures the convergence of the economy to the optimal steady state equilibrium \( E \). This dynamics would be precluded if \( N_0 > \bar{N} \). In this case, \( \nu \), the Lagrangian multiplier associated with the feasibility constraint, becomes strictly positive. Consequently, as depicted in Figure 3.1, the population will instantaneously adjust to \( \bar{N} \) along \( \Gamma(c, N) \) and thereafter converge on the saddle path \( \phi(c, N) \) to the steady state \( E \). To sum up, we can say that under the assumptions A1 and A2, the endogenous population is always bounded. If \( N_0 < \bar{N} \), the population dynamics follows the saddle path which converges monotonically to the optimal steady state equilibrium where the per capita consumption exceeds the subsistence level. If, however, \( N_0 > \bar{N} \), the convergence is not monotone.

We would also like to see how the degree of altruism \( \epsilon \) does affect the steady state equilibrium. Under the specification given earlier, \( \epsilon = 0 \) means that the parent attaches a weight equal to unity for their children’s utility regardless of the size of the family. The objective function (1) of the patriarch is Benthamite (sum-of-utilities). As \( \epsilon \) increases, this weight declines and the parent is said to be less altruistic. When \( \epsilon = 1 \), only the average utility of each child counts for the parent, and the objective function (1) of the patriarch is Millian (see Nerlove, Razin and Sadka (1986)). Consider now a decrease in the value of \( \epsilon \) within the open interval \( (0, 1) \) which amounts to an increase in the degree of altruism. Using again the phase diagram pictured in Figure 3.1, the locus \( \Phi \) given by (13) remains unchanged. However, the locus \( \Psi \) shifts down and to the right as one can obtain
from (12) $\partial c_i / \partial \epsilon > 0$ and $\partial N_i / \partial \epsilon < 0$. Thus the per capita consumption would be lower, and the population level in the long run higher, with an increase in altruism (and conversely). This is the so-called *Edgeworth conjecture* which clearly withstands in our model⁴.

In what follows, we shall call *poverty trap* a situation where the population size, despite possible technological progress as time unfolds, comes indeed ultimately to equilibrate with the subsistence level of consumption. The poverty trap is, indeed, a long run equilibrium, $S$, if $\hat{c} \leq c_s$ (see Figure 3.2). Thank to earlier discussion, the dynamics of the model in this case is quite clear. Note immediately that the relevant phase plan lies beyond the subsistence level $c_s$. The locus $\Phi(c, N) |_{\tilde{N}_i=0}$ intersect by now the locus $\Psi(c, N) |_{\tilde{N}_i=0}$ in its horizontal part at the level of $c_s$.

⁴In the context of endogenous growth, Razin and Yuen (1995) provide a quite interesting tradeoff between population growth and income growth. Instead of the level, it is the rate of population growth which is higher under Benthamite utility than under Millian utility.
Obviously, the convergence to $S$ can be carried out in two ways. If $N_0 > \overline{N}$, the convergent path sticks to $c_t = c_s$, $\forall t$. If $N_0 > \overline{N}$, this path would follow $\phi$ until it hits the consumption constraint, then reaches $S$ along the horizontal line $c_s c'_s$. Recalling that the steady state consumption is given by (19), we now state:

**Proposition 3.1.**

1. If the subsistence level of consumption, $c_s$, falls short of $\hat{c} = \frac{\beta(1-\sigma)(1+\sigma+\gamma)}{\sigma(1-\epsilon) + \sigma(1-\sigma)(\gamma-1)}$, the economy tends in the long run to an equilibrium level of consumption $c_\infty = \hat{c}$.

2. If the subsistence level of consumption, $c_s$, exceeds $\hat{c} = \frac{\beta(1-\sigma)(1+\sigma-\gamma)}{\sigma(1-\epsilon) + \sigma(1-\sigma)(\gamma-1)}$, the economy tends in the long run to the poverty trap.

**4. Escaping the Poverty Trap**

The question we now ask is whether the economy can escape from this poverty trap. The first element of hope for such an escape may come from possible technological progress. We distinguish, as usual, neutral from non-neutral progress. For the former kind of progress, marginal productivities of factors increase over time in the same proportion, leaving their productivity shares unchanged. For the latter, the technological progress favors one factor over the other, and the production shares of factors will be accordingly modified.

Let us first consider a neutral technological shift which, like manna from heaven, occurs once and for all at some time. The simplest way to take this into account is assigning a parameter $\tau$ to our production function $\tau AN_{\mu}^2$, where $\tau \in [1, \overline{\tau}]$. The technological shift will exert a horizontal translation of the loci $\Phi, \Psi$ as well as $\Gamma$ to the right (see Figure 3.2). Recalling that $\hat{\tau}$ and $\overline{N}$ are given by equations (17) and (18) for which $\tau = 1$, it is easy to check that $\hat{\tau}(\tau) = \hat{\tau}$, and $\hat{N}(\tau) = \tau^{1/1-\gamma} \overline{N}$. Thus, paradoxically, even in presence of neutral technological progress, the economy cannot escape from the poverty trap. This kind of progress will lead to a population increase while the per capita consumption remains unchanged: a neutral technological shift does not enable the economy to escape from a poverty trap.

However, it might be of interest to make this technological progress itself endogenous in the present model. Not as manna that fall from some rather obscure heaven, the technological advance is the fruit of R&D efforts which are by no
mean free to the society. In order to do so, let us now assume that the production function takes the following form:

$$F(T, N_t) = \left( A_t T \right)^{1-\gamma} (l_t N_t)^\gamma, \quad 0 < \gamma < 1,$$

where $A_t$ is the technological knowledge at $t$ which is now endogenously determined. Following Lucas (1988) we assume that, in order to increase $A_t$, the agent must now reduce by a factor $(1-l_t)$ the time he devotes to the production in order to engage in R&D activities. In this framework, equation (3) becomes

$$\dot{N}_t = \frac{1}{\beta} \left( A_t T \right)^{1-\gamma} (l_t N_t)^\gamma - \frac{1}{\beta} N_t c_t - \frac{I_t}{N_t} - N_t, \quad N_0 \text{ given.} \quad (3')$$

Moreover, we assume that

$$\dot{A}_t = \mu A_t [1 - l_t] \quad (21)$$

where $\mu$ is the marginal productivity of R&D efforts. It is easy to show that under this setting, although we have a constant growth of production, the population grows at the same rate and per capita consumption remains constant. Routine computation (see appendix) leads to

$$\phi = \frac{\beta (1 - \sigma) (1 + \rho - \gamma + (\epsilon - \gamma) \phi)}{\sigma (1 - \epsilon) + (1 - \sigma) (\gamma - \epsilon)} \quad (22)$$

where $\phi$ is the productivity growth rate. As previously, the per capita consumption converges toward

$$c_\infty = \begin{cases} c_s & \text{if } c_s > \check{c} \\ \hat{c} & \text{if } c_s \leq \check{c} \end{cases} \quad (19)$$

In our analysis, $\epsilon < \gamma$, so that in comparing (22) to (17), $\check{c} < \hat{c}$, and therefore $c_s > \check{c}$ implies $c_s > \check{c}$. Henceforth, if the economy is to reach the subsistence level of consumption, $c_s$, without a technological progress, it will do so even with an

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Footnotes:

5. The productivity growth rate is entirely determined by the parameters of the model. Routine computation leads to

$$\phi = \frac{\mu (\sigma - \epsilon) + \gamma^2 \left[ \sigma (1 + \rho - \mu) + \mu - \beta - \epsilon - \rho \right] + \gamma \left[ \epsilon (\beta + \rho - \mu) + \mu - \rho \right]}{\gamma^2 \left[ \beta - \sigma + \epsilon (2 - \sigma - \epsilon) \right] + \gamma \left[ 1 - \beta \epsilon - \sigma (1 - \epsilon) \right] + \sigma - \epsilon}$$

6. Note that if investment in land productivity is assumed away, (20) is identical to (15) as naturally expected.
endogenous neutral one. An endogenous neutral technological progress does not enable the economy to escape from a poverty trap.

What should be the nature of the technological progress allowing the economy to escape from the consumption subsistence level? We now consider a non-neutral technological progress which simply takes the form of a favorable change of the productivity of one factor relative to the other’s. Consider the kind of progress which leads to an increase of the marginal productivity of land relative to labor’s for any level of population fully employed in the economy. For simplicity, let us suppose a technological shift parameter $\tau$ such as $F (T, N, \tau) = T^{-\gamma(\tau)}N^{\gamma(\tau)}$. If this technological shift $\tau$ increases the marginal productivity of land relative to labor’s in the production, then by definition $\gamma' (\tau) < 0$. Using equation (17) and recalling that the denominator is positive, we easily obtain the function $\hat{c}(\gamma)$ which is strictly decreasing in the interval $[0, 1]$. It is straightforward that $\hat{c}(0) = \frac{\beta(1-\sigma)(1+\rho)}{\sigma - \epsilon}$. If $c_s \geq \hat{c}(0)$, it is impossible to escape from the poverty trap. Given $c_s$, we can define $\gamma(\tau) = \frac{\beta(1-\sigma)(1+\rho)}{(1-\sigma)(c_s-\beta)}$. A positive technological shift in land’s marginal productivity would decrease $\gamma$, thus bringing about an increase in the level of the steady state consumption, see Figure 4.1. To sum up:

**Proposition 4.1.**

1. A neutral technological progress does not enable the economy to escape from a poverty trap.

2. Consider a non-neutral technological shift which increases land’s marginal productivity relative to labor’s, i) if $c_s < \frac{\beta(1-\sigma)(1+\rho)}{\sigma - \epsilon}$ and ii) if the technological shift $\gamma(\cdot)$ is continuous in $\tau$ with $\gamma'(\tau) < 0$, then there exists a critical value $\tau$ beyond which the economy would escape from the poverty trap.

This proposition shows that only a non-neutral technological shift may help the economy to escape from the subsistence consumption level. Even captured by a poverty trap for some time, the economy will get out of the misery provided that sufficient non-neutral technological shift in increasing the productivity of land in the production could be realized.

Beside the technological progress discussed above, are there any other policy which might help the economy escaping from the miserable subsistence level of consumption? The answer to the above question naturally comes from public interventions through appropriately modifying the cost of child-rearing by a tax (subsidy) policy. Let $\theta$ denotes this policy instrument, with $\theta > 0$ if it is a child-rearing tax, and conversely. The net cost of child-rearing is therefore $\beta^\theta = \beta + \theta$. 

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Figure 4.1: Non-Neutral Technological Shift
All ingredients in this analysis can now be parametrized by $\theta$. Clearly both loci $\Psi$ and $\Phi$ shift to the left as $\theta$ increases, but the accessible frontier $\Gamma$ remains unchanged (see Figure 4.2).

Let us define $\Theta$ by

$$\Theta = c_s \frac{\sigma (1 - \epsilon) + (1 - \sigma) (\gamma - \epsilon)}{\beta (1 - \sigma) (1 + \rho - \gamma)} - \beta, \quad (21)$$

We are now ready to show

**Proposition 4.2.** Provided that $\Theta > \Theta$, it is possible to escape from the poverty trap through a child-rearing tax $\theta$.

This proposition is quite simple to understand. The introduction of a child-rearing tax discourage procreation by increasing the cost of a child. On the asset market, this would lower the net return to labor, thus having the same effect as a
decrease of labor’s marginal productivity relative to land’s as we have discussed previously. In this case, it is plain that rational agents will choose a path leading to a steady state equilibrium with a higher per capita consumption and fewer population than the subsistence trap outcome.

Before turning into our concluding remarks, it may be worth to relate our analysis to Galor and Weil (1998). In their model, two forces drive the transition between regimes. First, technological progress is driven both by increases in the size of the population and by increases in the average level of education. Second, technological progress creates a state of desequilibrium, which raises the return to human capital and induces parents to substitute child quality for quantity. In this section, we have shown that this disequilibrium effect can be induced by other things that lower the net return on children quantity namely a tax on child rearing or a non-neutral technological progress which increases the marginal productivity of land relative to labor.

5. Concluding Comments

In this paper, we have shown that, when fertility decisions are endogenous in a model where human beings constitute the sole capital asset, the economy may run into a poverty trap. However, we have demonstrated that the economy can escape from this unappealing long run situation through a suitable technological shift or an appropriate child-rearing tax. For the purpose of exposition, we adopt the Cobb-Douglas specification of the production function. But, all results withstand with a more general specification. The main assumption on the utility function which allows for making the poverty trap a possible equilibrium outcome is that the marginal utility at the subsistence level of consumption is finite. This is not hard to accept. However, if we abandon the form adopted in this paper for a more general utility function, we should expect to get multiple steady state equilibria, and consequently the study of the dynamics there involved should become much more demanding. Recent advances in the study of complex non-linear dynamics would suggest the possibility of cyclical behavior and even of chaos in such model. This issue seems to deserve a separate investigation.

Among others generalizations that we would like to mention, we first refer to the endogeneousization of the technological progress. One attempt is given in Amigues and Hung (1996) where one fraction of time that disposes each individual agent could be devoted to “education” of their children which then enhances the accumulation of human capital. In this case, population would grow without any
upper bound, a conclusion which is excluded in the present paper. There are of course many others avenues as well. Another aspect which seems to us quite interesting is to treat the carrying capacity, not as a fixed quantity of land as with the present model, but rather as an environmental stock which may be depleted with its use. One attempt is given in Makdissi (1999) where labor is combined to a renewable natural resource in order to produce the consumption good. In this case, if property rights are not well defined over the natural resource, the economy may converge toward a subsistence trap.
Appendix

When we allow the possibility for the agent to invest in land productivity, the problem becomes

\[
\max_{\{c_t, l_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} N_t^{1-\epsilon} c_t^{1-\sigma} dt
\]

subject to

\[
\dot{N}_t = \frac{1}{\beta} \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} - \frac{1}{\beta} N_t c_t - N_t, \quad N_0 \text{ given} \quad (\text{PA})
\]

\[
\dot{A}_t = \mu A_t (1 - l_t)
\]

\[
c_t - c_s \geq 0; \quad \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} - N_t c_t \geq 0
\]

Let denote by \( Y_t \) the total production at time \( t \) so that \( Y_t = \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} \). If \( \{c_t, N_t\}_{t=0}^{\infty} \) is the optimal path, then it satisfies Pontryagin conditions:

\[
N_t^{1-\epsilon} c_t^{1-\sigma} - \frac{\pi_t N_t}{\beta} + \lambda_t - \varphi_t = 0, \tag{A1}
\]

\[
\frac{\gamma \pi_t}{\beta} \frac{Y_t}{l_t} - \mu \psi_t A_t + \gamma \varphi_t \frac{Y_t}{l_t} = 0, \tag{A2}
\]

\[
\dot{\pi}_t = \rho \pi_t - \frac{(1 - \epsilon)}{(1 - \sigma)} \pi_t N_t^{1-\sigma} - \pi_t \left[ \frac{\gamma}{\beta} A_t N_t^{\gamma-1} - \frac{1}{\beta} c_t - 1 \right] + \varphi_t \left[ \gamma \frac{Y_t}{N_t} - c_t \right], \tag{A3}
\]

\[
\dot{\psi}_t = \rho \psi_t - \frac{(1 - \gamma)}{\beta} \pi_t \frac{Y_t}{A_t} - \mu \psi_t (1 - l_t) + \varphi_t (1 - \gamma) \frac{Y_t}{A_t}, \tag{A4}
\]

\[
\dot{N}_t = \frac{1}{\beta} \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} - \frac{1}{\beta} N_t c_t - N_t, \tag{A5}
\]

\[
\dot{A}_t = \mu A_t (1 - l_t), \tag{A6}
\]

\[
\lambda_t [c_t - c_s] = 0; \quad \lambda_t \geq 0; \quad c_t - c_s \geq 0, \tag{A7}
\]

\[
\varphi_t \left[ \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} - N_t c_t \right] = 0; \quad \varphi_t \geq 0; \quad \left( A_t L_t \right)^{1-\gamma} (l_t N_t)^{\gamma} - N_t c_t \geq 0. \tag{A8}
\]

where \( \pi_t \) represents the efficient price of capital and \( \psi_t \), the efficient price of technology \( A \). The solution path \( \{c_t, l_t, N_t, A_t\}_{t=0}^{\infty} \) satisfying (A1) to (A8) is optimal provided that

\[
\lim_{t \to \infty} e^{-\rho t} \pi_t N_t = 0, \tag{A9}
\]

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\[
\lim_{t \to \infty} e^{-\mu t} \pi_t \geq 0, \quad (A10)
\]
\[
\lim_{t \to \infty} N_t \geq 0, \quad (A11)
\]
\[
\lim_{t \to \infty} e^{-\mu t} \psi_t A_t = 0, \quad (A12)
\]
\[
\lim_{t \to \infty} e^{-\mu t} \psi_t \geq 0 \quad \text{and}, \quad (A13)
\]
\[
\lim_{t \to \infty} A_t \geq 0, \quad (A14)
\]

Assume for now that the two constraints are not binding \((\lambda_i = 0 \text{ and } \varphi_i = 0)\). Let \(l_\infty\) be the share of labor devoted to production on the equilibrium path. Equation (A6) becomes
\[
\dot{A}_\infty = \mu A_\infty (1 - l_\infty) = \phi. \quad (A15)
\]

Differentiating \(Y_t\) we get
\[
\frac{\dot{Y}_t}{Y_t} = (1 - \gamma) \frac{\dot{A}_t}{A_t} + \gamma \frac{\dot{l}_t}{l_t} + \gamma \frac{\dot{N}_t}{N_t}. \quad (A16)
\]

On the equilibrium path we have
\[
\frac{\dot{Y}_\infty}{Y_\infty} = (1 - \gamma) \phi + \gamma \frac{\dot{N}_\infty}{N_\infty}, \quad (A17)
\]

which is consistent with
\[
\frac{\dot{Y}_\infty}{Y_\infty} = \frac{\dot{A}_\infty}{A_\infty} = \frac{\dot{N}_\infty}{N_\infty} = \phi. \quad (A18)
\]

Let denote by \(\kappa\) the per capita production on the equilibrium path. From (A5) we know that on the equilibrium path we have
\[
\phi = \frac{\kappa}{\beta} - \frac{\hat{c}_t}{\beta} - 1. \quad (A19)
\]

Differentiating (A1) and substituting for \(\frac{\hat{c}_t}{\beta}\) we get
\[
-\sigma \frac{\dot{c}_t}{c_t} = 1 + \rho + \frac{(\epsilon - \sigma)}{\beta (1 - \sigma)} c_t \frac{\gamma}{\beta} \frac{Y_t}{N_t} + \frac{\dot{N}_t}{N_t}. \quad (A20)
\]
On the equilibrium path we have

\[ 0 = 1 + \rho + \frac{(\epsilon - \sigma)}{\beta (1 - \sigma)} \hat{c} - \frac{\gamma}{\beta} \kappa + \epsilon \phi. \tag{A21} \]

Using equations (A19) and (A21), we can solve for $\kappa$ and $c_\infty$

\[ \kappa = \frac{\beta (1 - \sigma) (1 + \rho + \epsilon \phi) + \beta (1 - \phi) (\sigma - \epsilon)}{\sigma (1 - \epsilon) + (1 - \sigma) \gamma}, \tag{A22} \]

\[ \hat{c} = \frac{\beta (1 - \sigma) (1 + \rho - \gamma + (\epsilon - \gamma) \phi)}{\sigma (1 - \epsilon) + (1 - \sigma) (\gamma - \epsilon)}. \tag{A23} \]

Differentiating (A2) and substituting for $\frac{\gamma}{\pi}$, $\frac{i}{\pi}$, $\frac{4}{\kappa}$, we get

\[ \frac{\pi}{\pi} = \frac{\psi}{\psi}. \tag{A24} \]

We can then use equation (A24) to solve for $\phi$

\[ \phi = \frac{\mu (\sigma - \epsilon) + \gamma^2 [\sigma (1 + \rho - \mu) + \mu - \beta - \epsilon - \rho] + \gamma [\epsilon (\beta + \rho - \mu) + \mu - \rho \sigma]}{\gamma^2 [\beta - \sigma + \epsilon (2 - \sigma - \epsilon)] + \gamma [1 - \beta \epsilon - \sigma (1 - \epsilon)] + \sigma - \epsilon}. \tag{A25} \]

Finally, remembering that we have the constraint $c_i \geq c_s$, we have

\[ c_\infty = \begin{cases} 
  c_s & \text{if } c_s > \hat{c} \\
  \hat{c} & \text{if } c_s \leq \hat{c} 
\end{cases}. \tag{A26} \]
References


