Estimating a Continuous-Time Asset Pricing Model
with State-Dependent Risk Aversion*

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Abstract

We propose a consumption-based capital asset pricing model in which the representative agent’s preferences display state-dependent risk aversion. Since a common factor – the state of the world – influences both stock prices and preferences, we obtain a valuation equation in which the vector of excess returns on equity includes both consumption risk as well as the risk associated with variations in preferences. We develop a simple model that can be estimated without specifying the functional form linking risk aversion with state variables. Our estimates are based on Markov chain Monte Carlo estimation of exact discrete-time parametrizations for linear diffusion processes. Since consumption risk is not forced to account for the entire risk premium, our results contrast sharply with estimates from models in which risk aversion is state-independent. We find that relaxing fixed risk preferences yields estimates for relative risk aversion that are (i) reasonable by usual standards, (ii) correlated with both consumption and returns and (iii) indicative of an additional preference risk of holding the assets.

Keywords: Asset pricing models, Bayesian analysis, continuous-time econometric models, data augmentation, equity premium puzzle, Markov chain Monte Carlo, risk aversion, state-dependent preferences.

JEL classification: C110, D810, G120

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1 Introduction

The mitigated empirical performance of the representative agent, consumption-based capital asset pricing model (C-CAPM) of Lucas (1978) and Breeden (1979) is well documented. If preferences are assumed to be state- and time-separable with an iso-elastic instantaneous utility function, and innovations are log-normally distributed, the model predicts that the vector of excess returns on risky over risk-less assets is affine in the covariance between consumption growth and returns (the quantity of consumption risk), with scalar loading factor given by the Arrow-Pratt coefficient of relative risk aversion (the price of consumption risk). Because of the observed weak correlation between consumption and equity, the high premia found in the data can only be replicated by setting risk aversion to implausibly high levels (the equity premium puzzle). Moreover, since for these models, risk aversion is also the reciprocal of the elasticity of inter-temporal substitution, the C-CAPM counter-factually predicts a large risk-free rate of interest to induce highly risk-averse agents to hold debt (the risk-free rate puzzle). Finally, excess returns are predictable, counter-cyclical, and strongly correlated with the price-dividend ratio; allowing for time-varying conditional covariance of consumption and returns is neither a sufficient nor a satisfactory response to the persistence found in the premia (the predictability puzzle, e.g. Cochrane, 1997, p.22).

To address these puzzles, one line of research emphasizes stochastic prices of consumption risks (among others Mark, 1988; McCurdy and Morgan, 1991; Harvey, 1991; Chou et al., 1992; Campbell and Cochrane, 1995; Bakshi and Chen, 1996). We follow this approach by relaxing the CRRA hypothesis of fixed curvature, and consider instead the alternative of state-dependent risk aversion (SRA). State-contingent preferences have been used successfully to explain behavior characterized by apparently excessive, or changing risk aversion. Robson (1992) reproduces ‘concave-convex-concave’ behavior of Friedman and Savage (1948) by incorporating preference over status to the instantaneous utility function; when status is allowed to depend on wealth distribution, richer curvatures may be obtained. Karni (1987, 1983) introduced state dependency

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1. Kocherlakota (1996); Campbell (1996); Cochrane (1997) provide excellent surveys of the current asset pricing puzzles and numerous attempts to solve them.
2. See Rustichini and Dreze (1994) for a review of state-dependent utility.
to rationalize the purchase of flight insurance by agents already holding life insurance. Since implausibly high (and potentially changing) abhorrence to risk is also found with standard asset pricing models, it seems reasonable to ask whether state-contingent preferences may be helpful in explaining the high returns on risky assets.

Several steps in this direction have already been taken. Observing that mean equity premia increase during recessions, Campbell and Cochrane (1995) consider a time-non-separable Hyperbolic Absolute Risk Aversion (HARA) specification in which the subsistence level is time-varying, a parameterization which can also be associated with a ‘slow-moving’ habit. As consumption falls toward subsistence during downturns, relative risk aversion increases, thus implying counter-cyclical attitudes toward risk. Building from Robson (1992), Bakshi and Chen (1996) also allow for status preference in the VNM utility function. To the extent that wealth-dependent self-perception of status influences marginal utility of consumption, the equilibrium relative risk aversion is a decreasing function of the individual’s wealth, and therefore also displays a counter-cyclical pattern.

We choose to address the issue of state-dependencies from a somewhat more general perspective: instead of specifying a functional form relating risk aversion to proxies for the state of the world, we treat risk aversion as a latent variable. In a companion paper (Gordon and St-Amour, 1998), we discuss how state-dependent curvature leads to rotations of the marginal utility schedule, which increase risks to the inter-temporal marginal rates of substitution (IMRS). Hence, if initial consumption is low and risk aversion is counter-cyclical, an unanticipated fall in consumption translates into a larger increase in marginal utility than under fixed preferences. Conversely, pro-cyclical curvature and high consumption lead to a larger decline in marginal utility following a positive innovation in consumption. These additional sources of risks affect the agent’s hedging decisions, and consequently the equilibrium returns on assets; excess returns then incorporate a second risk reflecting co-movements in risk aversion and returns (through the prevailing state of the world), which reduces the emphasis placed on consumption risk in explaining the high premia. In the special case where risk aversion and asset returns are not correlated, the model simplifies to one similar to that studied by Chou et al. (1992). The difference is not innocuous; we shall see that this second source
of risk plays a substantial role in explaining observed premia. Moreover, a natural explanation for time-varying excess returns is provided as the price of that second risk is a function of log consumption. Finally, precautionary savings arguments can be invoked to rationalize the low risk-free rate as a response to the increment in IMRS risk caused by time-varying curvature.

Our parameterization of time-varying risk aversion is both parsimonious and general: the concavity index is assumed to follow an Itô process. This assumption is useful as the joint process of curvature, log consumption and the integral measure of excess returns can be written as a multi-variate arithmetic Brownian motion. The advantage is that exact likelihood representations exist, and can be used to control for time aggregation, rather than using discrete-time approximation. We use Markov chain Monte Carlo (MCMC) and data augmentation techniques to estimate the SRA model using excess returns data. The main findings of this study are the posterior moments for time-varying risk aversion. These estimates are (i) moderately volatile, (ii) correlated with returns and consumption and (iii) well within the range of values that many consider to be reasonable (e.g. between 0 and 10). We find that the magnitude of the additional concavity risk is much larger than the one implied by consumption-returns covariances. Furthermore, the estimated posterior probability that the contribution of this risk is positive is roughly 90%, which would justify the high observed premia on risky assets without requiring high levels of risk aversion. Our results also suggest that risk aversion is negatively correlated with unanticipated consumption shocks, both in changes and levels; this is consistent with the counter-cyclical pattern to risk aversion that is a feature of the wealth-dependent specifications. Finally, our main findings are robust to re-scaling and portfolio choice.

This paper should be interpreted as a further empirical indication that at least some of the difficulties of the iso-elastic C-CAPM can be ascribed to its reliance on consumption risk as the sole justification for risk premia. Non-expected utility models do introduce additional covariances through the presence of gross returns on total wealth in the Euler equations, with favorable implications for estimated risk aversion parameters. However, the VNM restriction is seldom rejected when tested against non-expected utility models (e.g. Jorion and Giovannini, 1993). Furthermore, as pointed out by Kocherlakota (1996), the use of the market return on stock as a proxy for the return on total wealth clearly affects the estimated covariance
between equity returns and wealth and therefore the significance of this second source of risk. Wealthdependent specifications such as Bakshi and Chen (1996) also allow for these cross-correlations through the budget constraint, but their estimation is hindered by the absence of high frequency data on aggregate net worth. Our findings might be considered as unrestricted reduced-form evidence pointing in favor of such developments.\(^3\)

The paper is organized as follows. After the model is outlined in Section 2, our next objective is to identify the time series path of risk aversion indices consistent with post-war monthly US data. In Section 3, we estimate the SRA model by means of data-augmentation techniques, with results discussed in Section 4. Finally, a conclusion reviews the main findings, and offers suggestions for future research.

## 2 Asset Prices with State-Dependent Preferences

### 2.1 Theoretical Framework

Consider a representative agent who maximizes the discounted flow of future utility by allocating current wealth, \(W_t\) between consumption \(C_t\) and savings. Diversification of risk is obtained by selecting portfolio weights \(\omega_t = [\omega_{0,t}, \omega_{1,t}, \ldots, \omega_{n,t}]'\) between one risk-free asset (with subscript 0), and \(n\) risky assets. Conditional on state \(S_t\) occurring, the value of an asset at time \(t\) is denoted \(P_{j,t} \equiv P_j(S_t)\), for \(j = 0, 1, \ldots, n\). This price includes the value of all associated payments, such as dividends. Finally, the trading interval, i.e. the time elapsed between two consecutive trading periods, is denoted by \(h\).

The preferences of the agent are represented by state-dependent attitudes toward risk. The agent solves:

\[
J(W_t, S_t) = \max_{\{C_{t+h}, \omega_{t+h}\}} \mathbb{E}_t \sum_{\tau=0}^{T} \beta^{\tau+h} U[C_{t+h}, \gamma(S_{t+h})] \quad \beta \in (0, 1)
\]  

\(^3\)In fact, the SRA model can readily be adapted to allow for wealth dependency of consumption utility, in which case the representative agent’s problem becomes time non-separable. It is straightforward to derive conditions under which time consistency is verified.
subject to,

\[ W_{t+h} = (W_t - C_t) \sum_{j=0}^{n} \omega_{j,t} \frac{P_{j,t+h}}{P_{j,t}}, \quad \sum_{j=0}^{n} \omega_{j,t} = 1, \quad \forall t, \quad (2.2) \]

where \( E_t(\cdot) \equiv E(\cdot | I_t) \) denotes the conditional expectations operator with respect to information set \( I_t \).

Within-period (local) utility is assumed to be monotone increasing, strictly concave, and conditionally iso-elastic:

\[ U[C_t, \gamma(S_t)] = \Theta \frac{(\Theta^{-1} C_t)^{1-\gamma_t}}{1-\gamma_t}. \quad (2.3) \]

The constant preference parameter \( \Theta > 0 \) can be interpreted as a subjective scaling metric. The continuous, strictly positive state-dependent curvature index \( \gamma_t = \gamma(S_t) \) measures how rapidly marginal utility declines in consumption, and can be associated with the Arrow-Pratt measure of relative (consumption) risk aversion for a-temporal risk. Preferences (2.3) are a special case of the HARA utility function \( U(C) = \Theta[C/\Theta - \eta]^{1-\gamma}/(1-\gamma) \), with subsistence parameter \( \eta \) set to zero, and state-contingent concavity \( \gamma_t \).

The local utility function (2.3) captures the effects of pro- and counter-cyclical risk aversion on marginal utility risk, as outlined in Gordon and St-Amour (1998). Hence, for low consumption (with respect to \( \Theta \)), counter-cyclical risk aversion implies that for a ‘bad’ state, an unanticipated decline in consumption is associated with a clockwise rotation in the marginal utility schedule (with pivot \( C_t = \Theta \)) as \( \gamma_t \) increases, causing a larger increase in marginal utility. Conversely, for a ‘good’ state, pro-cyclical risk aversion and high consumption produce a larger decline in marginal utility following an unanticipated increase in consumption. This additional contributor to marginal utility risk will be shown to have important implications for the pricing kernels.

Next, following Hansen and Singleton (1996), let lower case letters denote logs, i.e. \( x \equiv \log(X) \), take the limit \( h \to 0 \) to obtain the continuous-time analog and assume that \( c(t) \), \( \gamma(t) \), and \( p_j(t) \) follow joint Itô processes. Then, using Itô’s lemma, it can be shown (Gordon and St-Amour, 1998, Appendix B) that mean excess returns and the risk-free rate follow:

\[ E(t)[dp_i(t) - dp_0(t)] = [c(t) - \theta] \sigma_{\epsilon i}(t) + \gamma(t) \sigma_{\epsilon i}(t) - 0.5 \sigma_{\epsilon i}(t) \quad (2.4a) \]
for risky returns $i = 1, \ldots, n$, and,

$$
dp(t) = -\log(\beta)dt + \gamma(t)E(t)[dc(t)] + [c(t) - \theta]E(t)[d\gamma(t)] + (1 - \gamma(t)[c(t) - \theta])\sigma_{\gamma c}(t)
- 0.5\gamma(t)^2\sigma_{cc}(t) - 0.5[c(t) - \theta]^2\sigma_{\gamma\gamma}(t)
$$

(2.4b)

for the risk-free rate, where $\sigma_{xy}(t) \equiv E(t)[dx(t)dy(t)]$ denotes the covariance between instantaneous changes in variables $x(t)$ and $y(t)$. The standard pricing equations for unconditional iso-elastic preferences are obtained by imposing $\gamma(t) = \gamma$, a constant in (2.4a) and (2.4b), which results in $E(t)[d\gamma(t)]$, and all covariances involving $\gamma$ being set to zero.

As discussed in Gordon and St-Amour (1998), SRA yields a higher equity premium if the contribution of concavity risk $[c(t) - \theta]\sigma_{\gamma i}(t)$ is positive. Note that the price of this risk is a function of log consumption, which could justify the time variation found in excess returns. Finally, a low risk-free rate of return is explained by additional volatility in IMRS's, through $\sigma_{\gamma c}(t)$ and also if the term $(1 - \gamma(t)[c(t) - \theta])\sigma_{\gamma c}(t)$ is negative.

### 2.2 The Parameterized Model

The econometric model is obtained by noting that (2.4a) is an arithmetic Brownian motion in the Itô vector $Y(t) \equiv [(p_i(t) - p_0(t)), c(t), \gamma(t)]^t$, $i = 1, \ldots, n$, for which exact likelihood representations exist.$^4$

Unfortunately, (2.4b) is quadratic and cannot be estimated jointly with (2.4a) through these methods. In consequence, the parameter $\theta$ is unidentified and is fixed to zero (i.e. $\Theta = 1$ in (2.3)) in the following analysis which will be based exclusively on (2.4a). As we show below, allowing for a different value of $\theta$ does not affect the main results.$^5$

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$^4$Here, $p_i(t) - p_0(t) \equiv \int_0^t [dp_i(\tau) - dp_0(\tau)]d\tau$ is the integral measure of excess returns.

$^5$Gordon and St-Amour (1998) estimate $\theta$ in price space, for a joint bond and equity prices specification in discrete time, and provide further evidence toward the robustness of the current results. Kocherlakota (1996) argues that the equity premium puzzle remains, by far, the most challenging of all empirical anomalies of the C-CAPM; the low risk-free rate is a puzzle only to the extent that risk aversion is excessively high.
Suppose for now that $\gamma(t)$ is an observable process. Then, the econometric model can be written as:

$$dY(t) = [AY(t) + b]dt + \sigma dZ(t)$$  \hspace{1cm} (2.5)

where $A = [a_{kl}]$ for $k, l = 1, \ldots, n$, $\gamma$ is a $n + 2$ square matrix; $b = [b_k]$ is a $n + 2$ column vector; $\sigma = [\sigma_k]$ is a $n + 2$ diagonal matrix; and finally $dZ(t) = [dZ_k]$ is a $n + 2$ vector of correlated Wiener processes, with $E(t)[dZ_k dZ_l] = \rho_{kl} dt$ and $\rho_{kl} \equiv \frac{\sigma_{kl}}{\sigma_k \sigma_l}$ is the correlation coefficient. Under these set of assumptions, the restrictions implied by the theoretical model are given by:

1. $a_{ic} = \sigma_{ic}$, \hspace{1cm} (2.6a)
2. $a_{i1} = a_{i2} = \ldots = a_{in} = 0$, \hspace{1cm} (2.6b)
3. $a_i = \sigma_{ic}$, \hspace{1cm} (2.6c)
4. $b_i = -0.5 \sigma_{ii}$, \hspace{1cm} (2.6d)

for risky assets $i = 1, \ldots, n$, while other parameters are left unrestricted. Substituting in equation (2.4a), the risk premium is

$$E(t)[dp_i(t) - dp_0(t)] = c(t)\sigma_{i\gamma} + \gamma(t)\sigma_{ic} - 0.5 \sigma_{ii}. \hspace{1cm} (2.7)$$

Note that the estimate of the risk aversion obtained under CRRA preferences will be biased upward if the contribution of the concavity risk $c(t)\sigma_{i\gamma}$ is positive. In the next section, we estimate the joint arithmetic Brownian motion (2.5) subject to the restrictions (2.6), accounting for time aggregation and the presence of unobserved concavity indices.
3 Empirical Methods

3.1 Evaluating the Likelihood Function

An attractive feature of the model described in (2.5) and (2.6) is its simplicity. However, applying the model to data is not straightforward: the process $\gamma(t)$ is not observed, and we do not have data available in a continuous-time format consistent with the model.

One approach for dealing with the fact the risk aversion is not an observed series would be to select a functional form for $\gamma(S(t))$, decide which state variables should be included in $S$ and impose the SRA restrictions; the resulting model would be expressed in terms of observed data. The drawback of this approach is its lack of generality: conclusions about the validity of the SRA framework would be conditioned on the ancillary hypotheses about the form and the arguments of $\gamma(S(t))$, as well as the laws of motion postulated for elements of $S$. Since the separable structure of (2.3) allows us to capture the role of the state variables in the single composite index $\gamma(t)$, and because we have as yet little guidance for modeling time-varying concavity, this study treats $\gamma(t)$ as an unobserved latent variable.

Regarding the second problem, since the observed data are not continuous processes, we make use of well-known results in the application of continuous-time models to time-averaged data. Suppose that $Y(t)$ satisfies (2.5) and define $Y_t \equiv \int_{t-1}^{t} Y(\tau) \, d\tau$. It can then be shown that the discrete time series process $\{Y_t\}$ satisfies

$$Y_t = \lambda Y_{t-1} + g + \eta_t$$

where $\lambda \equiv e^A$, $g \equiv (e^A - I)A^{-1}b$, where $e^A$ is the matrix exponential of $A$ and where the error vector $\eta_t$ is a multivariate MA(1) process (Melino, 1996; Grossman et al., 1987; Bergstrom, 1984; Phillips, 1976).

Note that (3.1) requires taking the inverse of the matrix $A$, and that $n^2$ elements are set equal to zero by the SRA restrictions (2.6). In principle, $A^{-1}$ exists for any $n$, but as $n$ increases, the determinant rapidly approaches zero. For parameter values in the range consistent with the data, reliable values for $g$ are only
available for the case where \( n \leq 2 \). It is for this reason that the empirical analysis below is limited to the \( n = 2 \) case. As we shall see, our results are quite robust to the choice of asset return pairs.\(^6\)

Let \( \gamma \equiv \{ \gamma \}_t^T \) represent the time series vector of time-averaged risk aversion coefficients from a sample of \( T \) observations; the initial value \( \gamma_0 \) is treated as a parameter. Let \( \phi \equiv \{ A, b, \Sigma \} \) denote the set of structural parameters. Given values for \( \eta_0, \phi, \gamma \) and \( \gamma_0 \), we can then evaluate the likelihood function \( L(\phi | \gamma, \gamma_0, \eta_0, \text{data}) \) according to the following procedure.

The error vector \( \eta_t \) in (3.1) is normally distributed, but these errors are not \( iid \); they follow a MA(1) process such that \( E[\eta_t \eta_t'] = \Omega_0, E[\eta_t \eta_{t-1}'] = \Omega_1, \) and \( E[\eta_t \eta_{t-s}'] = 0 \), for \( |s| > 1 \). Grossman et al. (1987) provide a straightforward way of calculating \( \Omega_0 \) and \( \Omega_1 \) as functions of \( \phi \).

It is well-known (Melino, 1996; Hamilton, 1994; Chib and Greenberg, 1994) that estimation of the parameters of moving average processes is greatly simplified when the model is characterized in terms of its state-space representation; our approach is based on the Bayesian procedure outlined by Chib and Greenberg (1994).

Since the error terms are not independent, the joint density of the sample cannot be expressed as the product of the marginal densities for \( \eta_t \). However, we can make use of the fact that \( \eta_t \) and \( \eta_{t-s} \) are uncorrelated for values of \( s \geq 2 \). Since the errors are normally distributed, the fact that they are uncorrelated means that \( \mathcal{P}(\eta_t | \eta_{t-1}, \eta_{t-2}, \ldots) = \mathcal{P}(\eta_t | \eta_{t-1}) \). These conditional distributions are also normally distributed, with conditional mean \( \hat{\eta}_t |_{-1} \equiv \Omega_1 \Omega_0^{-1} \eta_{t-1} \) and with conditional variance \( G^{-1} \equiv \Omega_0 - \Omega_1 \Omega_0^{-1} \Omega_1' \). Consider a sample of \( T \) observations generated by (3.1). The conditional distribution of \( \eta_1, \eta_2, \ldots, \eta_T \) given \( \eta_0 \) is therefore

\[
\mathcal{P}(\eta_1, \eta_2, \ldots, \eta_T | \eta_0) = \prod_{t=1}^{T} \mathcal{P}(\eta_t | \eta_{t-1}) \tag{3.2}
\]

\(^6\)We estimated the SRA model for all \( 10 \) possible pairs of assets, of which \( 4 \) were randomly selected for reporting purposes. The full results may be obtained from the authors upon request. We should also point out that this near-singularity problem would not be encountered in discrete-time models. However, it is also well-known that ignoring the time-aggregation problem poses extra problems of its own: see Heaton (1995, 1993). Moreover, restricting the dimension of \( Y \) to be no greater than 4 is not at odds with normal practice in estimating continuous-time models: Melino (1996) notes that over 97% of empirical applications are based on univariate or bivariate systems.
A problem is posed in the first period of the sample. Since data for $Y_{-1}$ are unavailable, there is no way to retrieve $\eta_0$; in order to facilitate the derivation of the likelihood function we suppose that $\eta_0$ is observed.

Given a value for the parameter vector $\phi$, we can compute the reduced-form parameters $\lambda$, $g$ and $G$ according to the procedures outlined above. From these parameters, we can then retrieve the sequence $\{\eta_t\}_{t=1}^{T-1}$ from (3.1). These values - along with $\eta_0$ - can then be used to compute the sequence $\{\hat{\eta}_{t|t-1}\}_{t=1}^T$.

Define $\varepsilon_t \equiv Y_t - \phi Y_{t-1} - g - \eta_{t|t-1}$ and let $\gamma \equiv \{\gamma_t\}_{t=1}^T$ represent the time series vector of time-averaged risk aversion coefficients. The unconditional joint density for the augmented data set is therefore

$$p_{\text{data}}(\eta_0, \gamma_0, \gamma, \phi) = \frac{(2\pi)^{-\frac{(n+1)T}{2}}|G|^{-\frac{1}{2}} \exp\{-0.5 \sum_{t=1}^{T} \varepsilon_t' G \varepsilon_t\}}{\prod_{t=1}^{T} p(\eta_t > 0|Y_{t-1}, \phi)}$$

(3.3)

The sample selection correction associated with imposing positivity for simulated values of $\gamma_t$ at each data point is taken into account by dividing (3.3) by the joint probability that a realized sequence of $\gamma$ is positive:

$$p_{\text{data}}(\eta_0, \gamma_0, \gamma, \phi, \gamma > 0) = \frac{(2\pi)^{-\frac{(n+1)T}{2}}|G|^{-\frac{1}{2}} \exp\{-0.5 \sum_{t=1}^{T} \varepsilon_t' G \varepsilon_t\}}{\prod_{t=1}^{T} p(\gamma_t > 0|Y_{t-1}, \phi)}$$

(3.4)

The augmented likelihood function is simply (3.4) interpreted as a function of the unknown parameters given the augmented data set.

### 3.2 Estimation Algorithm

We make use of Bayesian analysis to estimate our model. Much has been written (Zellner, 1971; Leamer, 1978; Poirier, 1988, 1995) on the theoretical justifications for doing so, and the recent development of Markov chain Monte Carlo techniques has greatly increased the feasibility of Bayesian methods of inference. In the current context, the fact that Bayesian methods are not obliged to make use of asymptotic theory makes them far preferable to the use of classical methods. The reason is that (2.7) implies that expected excess returns will depend on log consumption; if consumption is assumed to grow without bound, and if the processes $c(t)$ and $\gamma(t)$ are not co-integrated, then predictions for excess returns could be dominated by the $c(t)\sigma_c \gamma$ term. If this were the case, classical inference based on large-sample approximations would be problematic. On the
other hand, results from a Bayesian estimation are valid for any finite sample. The empirical results below suggest that our simple diffusion model is a reasonable approximation for our data,\(^7\) the model produces plausible forecasts that have no detectable trend.

If the risk aversion series were observed along with the vector \(\theta_0\), inferences about \(\phi\) would be based on the posterior distribution:

\[
P(\phi|\gamma, \gamma_0, \theta_0, \text{data}) = \frac{L(\phi|\gamma, \gamma_0, \theta_0, \text{data})P(\gamma_0)P(\phi)}{\int L(\phi|\gamma, \theta_0, \text{data})P(\gamma_0)P(\phi) \, d\gamma_0 \, d\phi}
\]

Two practical difficulties are posed by (3.5): \(\gamma, \gamma_0\) and \(\theta_0\) are not observed, and even if these values were observed, the non-standard form of the likelihood function suggests that evaluating the integral in the denominator of (3.5) looks to be a particularly daunting task. Two recent developments in the Bayesian statistical literature prove to be extremely useful in addressing these problems.

Tanner and Wong (1987) note that in many latent variable models, the estimation of the parameter vector is straightforward if the latent variables were observed; this is the case in the current setting. Their “data augmentation” approach is based on simulating values for the missing data from the model; the augmented data can then be used to estimate the fixed parameters. The applicability of data augmentation techniques - and Bayesian methods in general - has become markedly easier with the development of Markov chain Monte Carlo techniques. It is often the case that the application of MCMC with data augmentation is easier than trying to estimate latent variable models using classical techniques (McCulloch and Rossi, 1994; Jacquier et al., 1994).

\(^7\)Keeping in mind that the log of real per-capita consumption ranges from 8.0 to 8.4 in our sample, this assumption is not too farfetched.
In this application, the estimation is based on an iterative algorithm in which draws are made according to:

\[
\begin{align*}
\gamma^{(m)} & \sim \mathcal{P}(\gamma|\gamma^{(m-1)}, \eta_0^{(m-1)}, \phi^{(m-1)}, \text{data}) \\
\eta_0^{(m)} & \sim \mathcal{P}(\eta_0|\gamma^{(m)}, \gamma_0^{(m)}, \phi^{(m-1)}, \text{data}) \\
\gamma_0^{(m)} & \sim \mathcal{P}(\gamma_0|\gamma^{(m)}, \eta_0^{(m-1)}, \phi^{(m-1)}, \text{data}) \\
\phi^{(m)} & \sim \mathcal{P}(\phi|\gamma^{(m)}, \gamma_0^{(m)}, \eta_0^{(m)}, \text{data})
\end{align*}
\]  

(3.6)

Under fairly weak conditions that are satisfied in this application, \(^8\) the sequence of draws \(\{\phi^{(m)}, \gamma^{(m)}, \gamma_0^{(m)}, \eta_0^{(m)}\}\) forms an aperiodic and irreducible Markov chain whose stable distribution is the joint posterior distribution \(\mathcal{P}(\phi, \gamma, \gamma_0, \eta_0|\text{data})\) (Gelfand and Smith, 1990). Given a sample of \(N\) draws from the posterior, we can consistently estimate the posterior moments of the parameters of interest.

We make use of data augmentation in the first two steps in (3.6), and in both cases, the simulation is straightforward. Since (3.1) is linear in \(\gamma_t\) and \(\eta_t\), the full conditional distributions for each element of \(\gamma\) and of \(\eta_0\) are also normal. Consider the full conditional distribution \(\mathcal{P}(\gamma_t|\gamma_{-t}, \eta_0, \phi, \text{data})\), where \(\gamma_{-t}\) denotes the elements of \(\gamma\) other than \(\gamma_t\). Isolating the contribution of \(\gamma_t\) to the likelihood, we note that \(\mathcal{P}(\gamma_t|\gamma_{-t}, \eta_0, \phi, \text{data})\) is proportional to the kernel of a normal density. The first step of (3.6) therefore reduces to a sequence of draws from a univariate normal distribution.

The second step of (3.6) is done in a similar manner. We note that we can isolate the contribution to the likelihood of each element of \(\eta_0\), and that its full conditional distribution also has the form of a univariate normal; values of \(\eta_0\) are generated using the appropriate normal distributions.\(^9\) Similarly, if the prior distribution for \(\gamma_0\) is normal, then the full conditional distribution is also normal, with mean and variance given by well-known formulae (Poirier, 1995, p. 293).

Given the output of the previous steps, the structural parameters can be dealt with in a straightforward fashion. Since the conditional distribution \(\mathcal{P}(\phi|\gamma, \gamma_0, \eta_0, \text{data})\) is non-standard, we use the \(^{3}\)-Metropolis-

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\(^8\)Roberts and Smith (1994). The algorithm used in this study satisfies the sufficient condition that the full conditional densities used to generate new values for the chain have positive mass everywhere in the admissible region.

within-Gibbs' technique of simulating draws for each element of $\phi$ using the Metropolis-Hastings acceptance-rejection algorithm.\textsuperscript{10}

### 3.3 Prior Specification

The fixed parameters of (2.5) are only instrumental to our analysis; our main interest lies in estimating the coefficients of relative risk aversion. Because our focus is on determining what levels of risk aversion are consistent with the data, we prefer to use priors that are sufficiently diffuse so as to allow the form of the posterior distribution to be dominated by the likelihood function.

Since the parameters describe the diffusion process of a set of variables that are fairly smooth, we believe that the absolute values of the elements of $\phi$ will be relatively small. In specifying the priors for the unrestricted elements of $A$ and $b$, we use normal priors centered around zero. Prior beliefs about the elements of the Cholesky root of $\Sigma$ are also described by a diffuse normal distribution. Let the $i$ and $j$ denote the assets whose returns are being used to estimate the model. The prior means are chosen to be consistent with $\sigma_\epsilon = 0.01$, $\sigma_i = \sigma_j = 0.02$, $\sigma_\gamma = 0.5$, $\rho_{ci} = \rho_{ij} = 0.1$, $\rho_{c\gamma} = -0.5$, $\rho_{ij} = 0.5$ and $\rho_{i\gamma} = \rho_{j\gamma} = 0.5$. The choices for the prior means for $\sigma_\epsilon$, $\sigma_i$, $\sigma_j$, $\rho_{ci}$, $\rho_{ij}$, $\rho_{i\gamma}$ and $\rho_{j\gamma}$ are roughly consistent with the levels of observed volatility for excess returns and for consumption growth. In choosing priors for the parameters governing the diffusion process for $\gamma(t)$, we incorporate our belief that fluctuations in levels of risk aversion will be fairly small. We also expect that movements in risk aversion will tend to be negatively correlated with consumption growth. From (2.7), the usual CRRA model will provide estimates of $\gamma$ that are biased upwards if the term $\log(C)\sigma_{\epsilon,\gamma}$ is positive. In our data set, $\log(C)$ is always positive, i.e. $c_t > \theta$, and we believe that the estimates for risk aversion generated by the CRRA models are implausibly high; this suggests a positive correlation between excess returns and risk aversion. Lastly, the mean parameter for the prior distribution for $\gamma_0$ is set equal to unity. We believe that these values are reasonably close to the region of the parameter

\textsuperscript{10}See Tierney (1994) or Chib and Greenberg (1995) for a description. The computational burden is not particularly onerous: for each individual portfolio, the program that generated our main results took about eight hours to run on a workstation.
space where the likelihood function has mass. In order to attain local uniformity in this region, the prior standard deviations are set at the relatively large value of 10.

It is well known that, for the generic iso-elastic C-CAPM, positive risk aversion is a sufficient condition for the existence of a general equilibrium (Kocherlakota, 1990). Moreover, the Euler equations are sufficient to characterize this equilibrium when $\gamma(t) > 0$. On the other hand, the sufficiency of the valuation equations is less clear in the presence of negative realizations for $\gamma(t)$. Since it is the usual practice to use the Euler equations as a basis for an econometric model, the stochastic specification of the model should reflect this non-negativity condition, e.g., by means of exponential or quadratic transformations of $\gamma(t)$, or by imposing a reflecting barrier on the diffusion process. Unfortunately, these transformations generate non-linear models that do not belong to the class of specifications that yield closed-form expressions for the likelihood function for time-averaged data.

In light of this, our model can be viewed as an approximation to one that bounds the support of $\gamma(t)$. Instead of using a density that ensures that all possible values of $\gamma(t)$ are positive, we impose this restriction on the realized values for risk aversion. Given the equivalence in the way that the MCMC algorithm (3.6) treats both the parameters and the latent variables, this can be interpreted as imposing a prior belief that risk aversion is positive throughout the sample. This is implemented by simulating values of $\gamma_t$ from the positive tails of the normal distributions and by incorporating this truncation in evaluating the augmented likelihood function for $\phi$ as in (3.4).

4 Results

4.1 Data

Estimation for the SRA model is performed on the same US monthly series which have been used extensively in assessing the performance of various asset pricing models. The main sources of data are the CRSP tapes, for the returns series, and CITIBASE for the consumption, deflators and T-Bills variables. Our sampling
period is 1960:2 to 1992:7, for a total of 390 observations. Table 1 in the Appendix provides selected summary statistics for the various series.11

Consumption: The real aggregate consumption series is the personal consumption expenditures (constant 1987 $) on nondurable goods [CITI: GMCNQ]. The corresponding implicit price deflator [CITI: GMDCN] is used to obtain the real rates of return below. Per-capita consumption is obtained by dividing by the total population, including armed forces overseas [CITI: POP].

Risk-free rate: Following standard practice, we approximate the risk-free rate by the monthly rate on 90 days T-bills [CITI: FYGM3]. The real rate is obtained by subtracting the inflation rate computed from the nondurables deflator.

Risky returns: We use the CRSP returns for the 5 major industry groups, i.e primary (1), manufacturing (2), transportation (3), trade (4) and finance/services (5). Again, the returns are expressed on a monthly base, with real returns obtained by subtracting the inflation rate on nondurables.

4.2 Parameter Estimates

Although the Markov chain (3.6) will in principle converge from any starting value and using any proposing distribution for the Metropolis-Hastings algorithm, it is more efficient to make use of a few trial runs to identify the region in which the posterior has mass in order to select the appropriate spread for the proposing distributions. After this was done, 500 iterations were used to initialize the chain; the results presented below are based on the 2000 draws that followed.12 The numerical standard errors for the posterior means for the parameters were calculated using the method in Geweke (1992): the numerical errors for the estimates for $\gamma$ are roughly 1.2% of the estimated means, while the numerical errors for $\phi$ are generally around 0.5% of their estimated posterior means.

Our main results are summarized in Figure 1, which graphs the time series of the estimated posterior means of the coefficients of relative risk aversion for portfolio 1-2 (primary and manufacturing). The first

---

11To compute the excess price variable we simply accumulated excess returns, from a base observation of 0.
12Since the chain had started in the region identified in the initial runs, convergence was in fact quite rapid.
thing to note is that the estimate\textsuperscript{13} for the series $E[\gamma_t|\text{data}]$ fluctuates in a fairly narrow band around a sample mean of 0.32, which certainly appears reasonable by usual standards.

These estimates generated by our simple SRA specification are particularly encouraging. Comparatively small movements in risk aversion appear to be able to reproduce the large movements in IMRS suggested by observed levels of excess returns. Moreover, these fluctuations in marginal utility can be generated for levels of relative risk aversion that are generally much lower than estimates obtained by state-independent preference specifications.

Table 2 reports the posterior moments of the elements of $\phi$ and $\eta_0$, as well as the posterior moments for the standard deviations and correlation coefficients associated with $\Sigma$. The most noteworthy result

\textsuperscript{13}At each data point, we have 2000 draws for $\gamma_t$; the estimate for the sequence of posterior means is the sequence of averages of these artificial samples.
provided by Table 2 is the strong negative correlation between the innovations to consumption and relative risk aversion; the estimated posterior mean\footnote{Given 2000 time series for $\gamma_t$, we generate 2000 associated coefficients of correlation.} for the coefficient of correlation between $\Delta \gamma_t$ and $\Delta \alpha_t$ varies between -0.87 and -0.90. Intuitively, one might expect that time-varying measures of relative risk aversion would tend to be counter-cyclical; the results here provide strong evidence to support this conjecture.

In deriving mean excess returns, it was noted that the estimate of constant relative risk aversion would be larger than the one recovered under SRA if $c(t)\sigma_{t\gamma}$ is positive. Our results suggest that this is apparently verified in US data: the posterior probability that this term is positive varies between 89% and 96%. Moreover, the relative size of the consumption risk term $\gamma(t)\sigma_{ic}$ is quite small, typically less than one per cent of the concavity risk term’s contribution to the risk premium. Since consumption risk appears to account for only a small portion of the premia, it is probably unsurprising that the standard C-CAPM fares so poorly in empirical applications.

Earlier rounds of estimation suggest that our results are quite robust to variations in the specification of our priors. Choosing larger values for the prior standard deviation do not affect the results, and a round of estimation using an improper prior where $P(\phi)$ is proportional to an arbitrary constant generates results that are also similar to those reported above.

Note that the risk premium (2.4a) depends on the log of the consumption measure and that arbitrary changes in the definition of the units for consumption affect the value of $\log(C)$. It is natural to ask to what extent our results are robust to just such a re-normalization, i.e. other values of $\theta$. For this purpose, we performed a supplementary round of estimation in which $\theta = \log(10^5) = 11.5$ so that the mean of $\log(C_t)$ over the sample was -3.3. The estimates for relative risk aversion are reasonably close to those obtained above (mean of 1.25, standard deviation of 0.61) and exhibit a strong negative correlation with consumption growth ($\rho_{c\gamma} = -0.92$). The only important\footnote{Estimates for $A$ and $b$ are of course affected by this redefinition, but these parameters play no role in determining the risk premium.} change is that the estimate for the covariance parameter $\sigma_{t\gamma}$ adjusts so that the estimate for the term $\log(C)\sigma_{\gamma\gamma}$ is roughly the same as the estimate generated by our main results. This suggests that the product $\log(C)\sigma_{t\gamma}$ is identified, but not $\sigma_{\gamma\gamma}$. Notwithstanding this caveat, the
main empirical conclusions of this study are robust to rescaling the consumption series: both consumption measures generate similar estimates for risk aversion as well as similar estimates for the decomposition of the risk premium between consumption and concavity risk.

Our results are based on the exact likelihood function for time-averaged data, but it appears that they are also robust to minor variations in the specification for the likelihood function. In a round of estimation that used the mid-point approximation \( y_t = \frac{1}{2}(Y_t + Y_{t-1}) \) suggested by Bergstrom (1984), we obtained results that were similar to the ones recovered with the exact discrete-time model. Since continuous-time models are much easier to estimate using the mid-point approximation, this is an encouraging result for future work on the SRA model.

4.3 Forecasting Properties and Model Evaluation

Campbell and Cochrane (1995) – hereafter C-C – suggest that the changing slope of the mean-variance frontier can be successfully addressed by appending a time-varying subsistence level to the standard C-CAPM. This model yields time-varying risk aversion, in that the Arrow-Pratt measure is inversely proportional to the excess consumption over and above the non-stationary bliss point, with factor of proportionality given by the curvature of the consumption utility function. In their framework, relative risk aversion is counter-cyclical in the sense that it increases when consumption falls toward subsistence. As counter-cyclical risk aversion is also a potential feature of our model, C-C’s model is a natural alternative which we use as basis of comparison in evaluating our model’s performance.\(^{16}\)

\(^{16}\)The ‘self-perceived status’ of Bakshi and Chen (1996) model would also be a natural contender for SRA. However, as the authors point out, the absence of adequate data on aggregate wealth, as well as potential aggregation biases render estimation hazardous. Their first model (‘private wealth is status’) which they estimate does not allow for time-varying risk aversion. For this reason, we choose C-C’s model as the alternative. We also estimated the generalized recursive utility model of Epstein and Zin (1991) which yielded similar results as C-C.
4.3.1 Slow-Moving Habit Model

For within-period utility \( U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \), the excess return is:

\[
E \left( \frac{P_{i,t+1}}{P_{i,t}} - \frac{P_{0,t+1}}{P_{0,t}} \right) = -\frac{\sigma_{ij}}{2} + \gamma [1 + \lambda(s_t)] \sigma_{ie}. \tag{4.1}
\]

In this framework, \( \lambda(s_t) \), is a ‘sensitivity function’ which measures the covariance between log excess consumption \( s_t \equiv \log \left( \frac{C_t - X_t}{C_t} \right) \) and innovations in consumption growth.

The excess returns (4.1) form the basis for the alternative model. Assume that the laws of motion obey:

\[
\begin{align*}
\Delta c_{t+1} &= g + \nu_{t+1}, & \nu_{t+1} &\sim N.I.D \left(0, \sigma_{\epsilon \epsilon} \right) \\
 s_{t+1} &= (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) \nu_{t+1}.
\end{align*}
\tag{4.2}
\]

with scedastic structure

\[
\lambda(s_t) = \begin{cases} 
\bar{s}^{-1} \left[ 1 - 2(s_t - \bar{s}) \right]^{1/2} - 1, & \text{if } s_t \leq s_{\text{max}}; \\
0, & \text{otherwise},
\end{cases} \tag{4.3}
\]

where \( \bar{s} \equiv \left( \frac{\sigma_{\epsilon \epsilon} \gamma}{1 - \phi} \right)^{1/2} \) is the steady-state excess consumption, and where \( s_{\text{max}} \equiv \bar{s} + (1 - \bar{s}^2) \). Equations (4.2) and (4.3) are estimated jointly with (4.1). The parameters are \( \phi = [\gamma, \phi, g, Q_{c,c}, Q_{c,i}, Q_{c,j}, Q_{i,i}, Q_{i,j}, Q_{j,j}, s_1]' \); for \( i,j = 1, \ldots, 5 \), and where the upper-diagonal matrix \( Q \) is the Cholesky decomposition to the covariance matrix of the innovations. In addition, \( s_1 \) is the first element of the autoregressive process for log excess consumption \( s_t \).

4.3.2 Estimation Results

For comparison purposes, we also use the bivariate portfolios composed from the 5-assets returns matrix. The estimation strategy is similar to the one employed under SRA.\(^{17}\) The results are reported in Table 3.

\(^{17}\)More specifically, we use \( \sigma_c = 0.01, \sigma_i = \sigma_j = 0.02 \), with \( \rho_{c,c} = \rho_{i,c} = 0.10 \) in order to construct prior elements of \( Q \). In addition, we use the calibrated elements in C-C for \( \gamma = 2.37, \phi = 0.970 \), and \( s_1 = \log(0.049) \). The prior for consumption growth is set at \( g = 0 \). Finally, as before, a standard error of \( 10 \) is postulated for all prior elements. The posterior means are based
A number of elements stand out from the analysis of Table 3. First, note that the structural parameter estimates for $[\gamma, \phi, g, s]$ are remarkably robust across portfolios. Secondly, the point estimates reflect some differences when compared to C-C’s calibrated parameters. The curvature parameters $\gamma$ are all higher than C-C’s assumed level of 2.372, although not significantly different. In addition, the AR(1) coefficient in the univariate process for log excess consumption growth, $\phi$, is on average lower at 0.55 than C-C’s assumed 0.97. Mean excess consumption $S_t \equiv \frac{C_{t+1} - X_t}{C_t}$ is centered at 2.9%, reasonably close to C-C’s steady-state value of 4.9%. The estimated average risk aversion however is centered around 115, a value higher than C-C’s reported 48.4 and certainly outside any reasonable range for relative risk aversion. Moreover, the estimates are relatively precise, with standard errors of the order of 3.2. This suggests that the ‘slow-moving habit’ model of C-C is apparently unable to generate coefficients of relative risk aversion in the ranges deemed acceptable.

The reason for this is that although, by construction, risk aversion is counter-cyclical in C-C’s model, concavity risk is not present. As consumption falls toward a slow-moving subsistence level, risk aversion increases. However, any covariance with asset returns is obtained through the consumption covariance. By not allowing any independent covariance (which is obtained through the joint dependence of returns and risk aversion on states of the world under SRA), the slow-moving habit model does not in any way alleviate the problems associated with the smooth consumption series relative to the volatile returns.

4.3.3 Comparing Forecasting Properties

Since the C-C and SRA models are not nested, a comparison of their forecasting properties is a natural criterion for model evaluation. Towards this end, we performed one-step-ahead forecasting exercises for both models over the last 24 months of the sample for portfolio 1-2. For each period, we computed the posterior predictive moments for $Y_{t+1}$ conditional on period-$t$ information; these estimates take into account the posterior uncertainty about the parameters as well as the risk aversion series. In addition, we also computed

\[ \text{on every } 5^{th} \text{ element of } 10,000 \text{ draws. Due to the auto-regressive structure inherent in C-C’s model, we do not incorporate corrections for time-aggregation.} \]
the predictive densities for each data point in order to perform a formal statistical comparison of the two non-nested models.\footnote{Each forecasting exercise involved a round of estimation based on data observed up to period $t$. Since over 90\% of the sample is retained in the estimation, these results should be relatively robust to the choice of prior. For more on the use of predictive densities in model evaluation, see Geweke (1994).}

**Figure 2: Forecasted and actual excess returns for manufacturing.**

As can be seen in Figure 2, both models are capable of reproducing the unconditional mean of excess returns, but the SRA model is clearly better at reproducing the fluctuations in the realized series. Although the conditional first moments implied by C-C are time-varying, the movements in the one-step-ahead forecasts
are dwarfed in comparison with observed volatility. Moreover, observe that no trending component can be
detected from the SRA forecasts, even though the estimated model allows for non-stationarity in predicted
excess returns. Despite the strong simplifying assumptions that were made in specifying the economic model
and in the statistical data-generating process, our SRA model is capable of producing surprisingly good forecasts.

Finally, when SRA models and C-C are compared, the results favor the C-C model with an estimated log
Bayes factor of 14.2. This result is not surprising given that Bayesian model comparison exercises tend to
favor more parsimonious models, and that modeling \( \gamma(t) \) as an unobserved latent variable adds considerable
uncertainty to our forecasts. Presumably, specifications that parameterize \( \gamma(S(t)) \) would augment the fore-
casting precision of the SRA model. Furthermore, recall that the C-C model requires values for relative risk
aversion in the neighborhood of 100 in order to reproduce the premia, whereas the SRA model’s estimates of
around 0.3 are more plausible. Moreover, the square roots of the mean squared errors (RMSE) for the C-C
model are not dramatically lower: the RMSE for the return on assets 1 (primary) and 2 (manufacturing)
are 0.043 and 0.046, compared to 0.049 and 0.047 for the SRA. Based on these elements, we conclude that
our results are encouraging and that they deserve further investigation.

5 Conclusion

The application of state-dependent preference specifications to the C-CAPM is motivated by a long-standing
paradox of conventional macro-economic asset pricing models: since asset returns and consumption are
weakly correlated, observed equity premia can only be generated by an unreasonably high risk aversion
index. In the SRA framework, observed risk premia can be replicated with plausible levels of relative risk
aversion indices. The reason for this is the presence of an additional concavity risk that supplements the
usual consumption risk within the valuation equation. Moreover, the price of this risk is a function of
log consumption, providing a rationale for time-varying and counter-cyclical excess returns. Finally, the
increment in IMRS risk obtained under SRA justifies a low risk-free rate of return through a precautionary savings argument.

Our empirical implementation concentrates on identifying both the characteristics of the stochastic process for risk aversion consistent with US data as well as the predicted values of this process. After specifying a simple linear multivariate diffusion system, we use data augmentation techniques to provide estimates for their realized values at each data point. In resorting to Bayesian methods, we provide finite-sample results based on the exact likelihood function for time-averaged data. The estimated posterior moments point to a sequence of relative risk aversion indices that are (i) considerably lower in mean than their estimated state-independent counterparts, and (ii) correlated with returns and consumption. This last finding suggests a counter-cyclical pattern to risk aversion, as well as a positive contribution of preference risks.

Our results provide further evidence that the C-CAPM model with iso-elastic separable utility is unable to account for non-consumption covariances between returns and the stochastic discount factor. Admittedly, this does not constitute an answer to the equity premium puzzle to the extent that few theoretical restrictions are imposed. On the other hand, the fact that they are absent implies that our estimated process for risk aversion is not contaminated by faulty restrictions regarding functional forms or choice of proxies. As such, the process that we estimate could reasonably be considered as an unrestricted reduced form for time-varying risk aversion. Since so little is known about what factors influence our attitudes toward risk, it is felt that the simple model used here and its encouraging results warrant a second look at the fixed risk preferences hypothesis.
### A Tables

#### Table 1: Sample Statistics

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<th>correlation coef.</th>
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Note: \(\Delta p_{i,t}\) is net rate of return for industry groups \(i = \text{primary (1), manufacturing (2), transportation (3), trade (4) finance/services (5)}\). \(\Delta p_{0,t}\) is the net risk-free rate of return.
Table 2: Estimated parameters, SRA

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<td>(0.0026)</td>
<td>(0.0022)</td>
<td>(0.0018)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>0.774</td>
<td>0.800</td>
<td>0.728</td>
<td>0.850</td>
</tr>
<tr>
<td></td>
<td>(0.0209)</td>
<td>(0.0179)</td>
<td>(0.0241)</td>
<td>(0.0141)</td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>0.0225</td>
<td>0.0196</td>
<td>0.0289</td>
<td>0.0219</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.0167)</td>
<td>(0.0292)</td>
<td>(0.0134)</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>0.0600</td>
<td>0.0491</td>
<td>0.0713</td>
<td>0.0637</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.0018)</td>
<td>(0.0026)</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>$\rho_{ji}$</td>
<td>0.0242</td>
<td>0.0187</td>
<td>0.0270</td>
<td>0.0239</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.0155)</td>
<td>(0.0289)</td>
<td>(0.0137)</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>0.255</td>
<td>0.310</td>
<td>0.114</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td>(0.0145)</td>
<td>(0.0223)</td>
<td>(0.0051)</td>
<td>(0.0148)</td>
</tr>
<tr>
<td>$E(\gamma_t)$</td>
<td>0.324</td>
<td>0.415</td>
<td>0.168</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>(0.0606)</td>
<td>(0.141)</td>
<td>(0.0363)</td>
<td>(0.0925)</td>
</tr>
<tr>
<td>$STD(\gamma_t)$</td>
<td>0.0569</td>
<td>0.0810</td>
<td>0.0327</td>
<td>0.0831</td>
</tr>
<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.0159)</td>
<td>(0.0052)</td>
<td>(0.0119)</td>
</tr>
</tbody>
</table>

Note: Note: Standard errors in parentheses. Portfolios including assets $i$ and $j$ are denoted 'port $i - j$', where industry groups are primary (1), manufacturing (2), transportation (3), trade (4) finance/services (5). $E(\gamma_t)$ average of 2000 draws x 390 observations of relative risk aversion; $STD$ is average of standard errors.
Table 3: Estimated parameters, Habit Persistence

<table>
<thead>
<tr>
<th>params</th>
<th>port 1-2</th>
<th>port 2-3</th>
<th>port 3-4</th>
<th>port 4-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>4.84</td>
<td>5.73</td>
<td>6.16</td>
<td>5.76</td>
</tr>
<tr>
<td>(4.31)</td>
<td>(5.21)</td>
<td>(5.06)</td>
<td>(4.99)</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.554</td>
<td>0.572</td>
<td>0.541</td>
<td>0.559</td>
</tr>
<tr>
<td>(0.406)</td>
<td>(0.376)</td>
<td>(0.392)</td>
<td>(0.394)</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>8.70e-04</td>
<td>9.13e-04</td>
<td>8.89e-04</td>
<td>8.77e-04</td>
</tr>
<tr>
<td>(3.64e-04)</td>
<td>(3.79e-04)</td>
<td>(3.77e-04)</td>
<td>(3.90e-04)</td>
<td></td>
</tr>
<tr>
<td>$s_1$</td>
<td>-1.18</td>
<td>-4.50</td>
<td>-5.63</td>
<td>-3.37</td>
</tr>
<tr>
<td>$(8.76)$</td>
<td>$(9.35)$</td>
<td>$(9.57)$</td>
<td>$(9.62)$</td>
<td></td>
</tr>
</tbody>
</table>

| $\sigma_c$ | 0.0073 | 0.0073 | 0.0074 | 0.0074 |
| (0.0003) | (0.0002) | (0.0003) | (0.0003) |
| $\rho_{c,i}$ | 0.123 | 0.113 | 0.108 | 0.120 |
| (0.0489) | (0.0450) | (0.0436) | (0.0423) |
| $\rho_{c,j}$ | 0.119 | 0.112 | 0.112 | 0.116 |
| (0.0497) | (0.0437) | (0.0446) | (0.0415) |
| $\sigma_i$ | 0.0466 | 0.0464 | 0.0379 | 0.0568 |
| (0.0017) | (0.0017) | (0.0014) | (0.0020) |
| $\rho_{c,j}$ | 0.762 | 0.790 | 0.720 | 0.847 |
| (0.0218) | (0.0192) | (0.0255) | (0.0142) |
| $\sigma_j$ | 0.0536 | 0.0378 | 0.0568 | 0.0501 |
| (0.0019) | (0.0013) | (0.0021) | (0.0018) |
| $E(RRA)$ | 108 | 117 | 120 | 116 |
| (3.24) | (3.31) | (3.25) | (3.17) |
| $STD(RRA)$ | 65.8 | 67.4 | 65.7 | 64.4 |
| (1.84) | (1.76) | (1.83) | (2.14) |

Note: Standard errors in parentheses. Portfolios including assets $i$ and $j$ are denoted by $\text{port } i-j$, where industry groups are primary (1), manufacturing (2), transportation (3), trade (4) finance/services (5). $E(RRA)$ average of 2000 draws x 390 observations of relative risk aversion; $STD$ is average of standard errors.
References


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