FAMILIES, INSURANCE AND EMPLOYMENT
IN DEVELOPING AGRICULTURAL ECONOMIES

by

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1 Introduction

In many less-developed countries (LDCs), elders and adults pool resources and risks through the extended family – henceforth family – in order to smooth-out lifetime consumption paths. The World Bank (1994, Table 2.4, p. 64) reports that 84% of old people in low-income economies lived with their children or family, while only 16% lived either alone or with spouse. Comparatively, these numbers are respectively 23% and 77% in high-income countries.

Affective ties are a plausible factor explaining why adults and elders live in a family. However, other elements matter. It is often argued that while elders in developed economies live on their savings or public transfers, those in developing countries enjoy inter-generational transfers.\(^1\) The World Bank study mentions that 58% of those over 65 expect to receive income from family members in low-income countries.\(^2\) In the Philippines, 67% of mothers over 65 anticipate financial contribution from their sons, and 88% expect them to help in emergencies. Lee, Parish and Willis (1994) report that in Taiwan, 79% of sons and 70% of daughters gave gifts to their parents while only 14% of sons and 21% of daughters received gifts from their parents. In Western Kenya, Hoddinott (1992) finds that absent sons provided the most transfers of money and goods to elderly parents. By comparison, in the US, only 29% of mothers and 19% of fathers expect financial contribution from their sons. This is confirmed by Lee and Miller’s (1994) findings that gifts and transfers in the US flow predominantly from elders to adults.

One motive for these intra-household flows is the sheltering role of the family. In LDCs, the family can be seen as an institution providing contingent-claims contracts against risks faced by its members. In developing agricultural economies, these risks arise for two reasons: i) unemployment, when agricultural labor wages are fixed at subsistence levels,\(^3\) and ii) crop failure, particularly for small-scale mono-crop cultivation. When complete markets for contingency claims are inaccessible in rural areas, informal structures such as *tontines*

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\(^2\) This number falls to 32% for middle-income countries ([1994, Table 2.3, p. 63]).

\(^3\) Mazumbar (1989, p. 10) concludes that empirical studies of wage determination in agriculture rarely explain more than 12% of the wage variance. As mentioned by Stiglitz (1986, p. 263), “... there appears to be involuntary unemployment, a phenomenon which seems inconsistent with the classical competitive models.” Moëne (1992) and Grossman (1994) fail to capture this stylized fact, essentially because they assume perfect competition between landowners.
in Central and West Africa emerge as risk-sharing mechanisms. Authors such as Pollak (1985, p. 584) argue that the family belongs to this set of risk-pooling institutions, and has provided “protection against the economic consequences of uncertain adverse effects” throughout history. More than merely being a passive recipient, the head of the family (usually the elder) directly supervises income distribution, to some extent, assuming the role of a benevolent dictator:

In a great majority of traditional cultures, a number of nuclear families act as an aggregate in their social and economic pursuits. [This composite or extended family] provides shelter and food for all its members, regardless of their individual contributions, so that the indigent and indolent alike are cared for in a sort of “social security” system. Working members are expected to pool their earnings for the benefit of everyone. The behavior and careers of its members are the close concern of the elders [Kerr, Dunlop, Harbison and Myers (1964, p. 67)].

Beyond cultural and peer pressure arguments, economic theory provides some explanation for the family institution. Becker and Barro (1988) and Barro and Becker (1989) were among the first to relate child-bearing decisions to lifetime inter-dependent utility, in which altruistic adults decide on the number of children, taking into account the rearing costs and the effect of their dependent’s consumption on their own utility. Upward transfers from adults to elders have been linked to a ‘setting-the-example’ effect by Cox and Stark (1992) and Bergstrom and Stark (1993), in which adults supporting elders hope to induce similar behavior from their children. Bergstrom (1996) also mentions the interesting possibility of an ‘helpers-at-nest’ effect, whereby once educated, adults must cover the education expenses of their younger siblings who remain at home under the supervision of elders. Conversely, downward transfers from parents to children have been explained by natural selection by Kaplan (1994), where transfers maximize the offsprings’ chances of viability, and to a ‘strategic bequest motive’, by Bernheim, Shleifer and Summers (1986, 1985), where elders exchange valuable contacts with their children with promises for eventual inheritance.

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4See Bergstrom (1994, 1996) for reviews.
In spite of these advances, many questions remain unanswered with respect to families. In particular, standard theory usually assumes that at least one generation (often children or elders) is not engaged in productive activities, a postulate which seems ill-suited for LDCs. Why an adult and an elder, both earning extra-family revenues, agree to transfer a portion of that revenue to one another rather than living separately and consuming their entire income is not considered in the literature. In particular, the ‘paying’ cohort is usually passive in the sense that relative bargaining strength is not analyzed. Furthermore, the transfers to the recipient cohort are often fixed and independent of prevailing economic conditions.

It may also be argued that theory provides no compelling explanation as to why the scope and direction of inter-generational flows should be related to the level of economic development. Moreover, the macro-economic allocative implications of the family in LDCs have not been developed. Since families are predominant in rural societies, a natural question concerns their impact on land and labour allocation. The unitary approach, which is most often used, assumes a unique set of preferences characterizing the household. As such, it abstracts from how income distribution among family members affects their consumption and labor supply decisions.

In light of these elements, this paper asks the following questions with respect to LDCs: i) why do families form and in particular, do families depend on the level of economic activity? and ii) does family organization affect the economic conditions? Tackling these two issues in a unified framework implies a feedback mechanism: the formation of families could depend on prevailing economic conditions which could in turn be influenced by the nature of the family organization.

To answer these questions this paper defines a family as a contractual arrangement whereby a member of a risk-averse cohort is insured by a member of the other less risk-averse cohort. The direction and magnitude of intra-family transfers depend on the relative bargaining position of each family member, their risk aversion, and importance of the risk. If the insurance premium payment depends on the outcome, shirking may occur.

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5The World Bank (1994, p. 94) reports that elders’ labour force participation rates in LDC’s are 5 times those found in industrialized countries.
6An exception is Becker and Barro (1988) who use a dynastic utility framework to show that fertility declines with the provision of social security.
Through the contiguity of members it entails, the family can be considered as one institution through which enforcement costs are minimized with the result that a family is an efficient risk-pooling agreement when moral hazard may occur.\(^8\)

We assume that a specific individual can insure another one and that positive expected rent extraction provides a rationale beyond altruistic motives for the formation of families. The family insurance contract is therefore not actuarially fair. That altruistic individuals may benefit from insuring other family members against adverse shocks has already been analyzed by Becker (1991). However, to our knowledge, that families may form purely for insurance services rent extraction motives has not been formally considered in the literature.

We consider consumption risk arising from two elements. First, adults may be hired as unskilled labour, at a fixed subsistence wage by an infinitely-lived unique landlord. As unskilled labour markets may not clear, a risk-averse adult may demand unemployment insurance (UI) services from an elder. Secondly, after receiving on-the-job training, a subset of the employed adults – now elders – receive state-contingent tenancy contracts from the landlord. They may farm plots of land rented from the landlord with production subject to adverse technological shocks. Faced with these risks, the risk-averse elders may seek crop failure insurance from an adult. In both cases, signing an insurance contract implies the formation of a family.

The landlord, who is a monopsonist in the skilled labour market, extracts all the sharecroppers’ rent. He appropriates whatever rent the insurer extracts from the insuree via the family. Since the price of unskilled labour is fixed, relative input prices are affected. Hence the landlord’s hiring decisions depends on the family structure. As family structure is endogenous, we show that for positive enforcement costs of contracts when the adult is insured by the elder, families disintegrate if the adult is weakly averse to risk and the unemployment risk is too high or too low. Under rational expectations, the landlord takes this switching property into account when designing tenancy rates.

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\(^8\)Binswanger and Rosenzweig (1986) investigate how family can be an optimal response to heterogeneous types of agents and asymmetric information between farm labour and employers. By hiring family members, an employer minimizes shirking and is more likely to know their competence.
We establish that families can be skilled-labour augmenting. Higher risk aversion or a decrease in enforcement costs increases the steady-state number of skilled sharecroppers, while reducing the number of unskilled laborers. Furthermore, the family switching property can yield multiple equilibria. We give an example of two equilibria: one with adult unemployment and skilled employment and the other with full adult employment and no skilled employment.

The paper is organized as follows. Section 2 describes the village economy, with optimal family and sharecropping contracts discussed in Section 3. We then present the landlord’s choice of skilled and unskilled labour in Section 4 where we perform comparative statics exercises. In Section 5, we numerically analyze the profit function and the potential for multiple equilibria. Finally, a conclusion reviews the main findings, as well as potential for further research.
2 Model

2.1 Village Organization

Consider a village in autarky populated by:

- 1 infinitely-lived landlord who owns all farmland of size $R$,
- $N$ landless adults with a two-period life horizon,
- $N$ landless elders with a one-period life horizon.

Labour markets are determined locally and there are no capital or land markets. A unique non-storable consumption good is produced by combining land and labor either $i)$ under the direct supervision of the landlord, which we define as farming, or $ii)$ the landlord can subcontract this operation to landless agents, defined as sharecroppers. The latter mode of production requires skills which are only acquired by working for the landlord as a labourer for one period. As landless agents live for two periods, only elders may receive a sharecropping offer. These sharecroppers can be seen as unsupervised agents working for the principal (i.e. the landlord).

A sharecropping contract stipulates the state-contingent landlord's and elders' output shares. Special cases of this arrangement are land tenancy, where the rent paid by the sharecropper is state-independent and pure sharecropping in which a state-independent share of realized output is paid to the landlord. As sharecropping usually takes place on small plots of land we assume fixed proportions technology:

$$ y = \min[r_s, \beta \times 1] $$

where $y$ is the sharecropper’s output, $r_s$ is the plot of land he farms and $\beta$ denotes labour productivity. The elder supplies one unit of labour, demands $\beta$ units of land, and consequently produces $\beta$ units of the consumption good. Hence a total of $S_t$ sharecroppers farm $\beta S_t$ units of land, leaving a maximum of $r_t = R - \beta S_t$ to the landlord for farming production.
The landlord hires labourers to work under his direct supervision while bearing in mind that he will need some of them to work as sharecroppers in the next period. For this reason, we impose that only adults can be hired as labourers in order to simplify the analysis. Hence, the number of labourers in \( t \) can be redefined as the sum of next period’s sharecroppers \( S_{t+1} \) and those labourers who receive no sharecropping offer \( L_t \).

The landlord trains all labourers as there is zero cost to training. Moreover, all labourers have identical contemporaneous productivity and are hired at a fixed efficiency wage \( W \). Since this wage is exogenous, adult unemployment can occur if the landlord’s demand for labourers is less than the adult population.

The landlord uses a quasi-concave production technology to combine \( L_t + S_{t+1} \) units of labour with \( R - \beta S_t \) unit of land to produce \( Y_t \) units of consumption goods:

\[
Y_t = F(L_t + S_{t+1}, R - \beta S_t) .
\]  

(2.2)

As in clear from the production function (2.2) the landlord faces an intertemporal trade off: training more labourers to be sharecroppers in the next period, reduces his next-period farming output by reducing available farming land \( r_t \), if everything else stays constant.

### 2.2 Preferences

All landless individuals maximize their expected utility, while the landlord who is more akin to a firm and is infinitely-lived, maximizes his present value expected profits. We assume that within a cohort, all landless individuals have identical preferences but that adults and elders differ in their attitudes towards risk. More specifically, an adult can be less averse to risk than an elder, or vice-versa. Following the characteristics of the village economy described in the previous sub-section, assume that landless individuals derive satisfaction from the quantity of goods they consume.

Hence, a representative individual utility within a cohort is a function of his consumption \( c \):

\[
U(c; \gamma_i) = c^{\gamma_i} .
\]  

(2.3)
for $\gamma \geq 1$, $i = \text{adult (a) or elder (o)}$, and indexing the preference parameter by the cohort captures the fact that preferences differ across cohorts. The preferences (2.3) can be associated with ‘moderate’ risk aversion. The Arrow-Pratt coefficient of relative risk aversion $1 - 1/\gamma_i$, is bounded above by 1 as $\gamma_i$ tends to infinity, such that (2.3) becomes:

$$U(c; \infty) = \log(c).$$

In an economy without any uncertainty any individual would choose his consumption level to maximize his utility. However, in our village economy, agents must decide without complete information, which we discuss next.

### 2.3 Information and uncertainty

Agriculture can be considered as risky. Adverse weather conditions, pests and other unpredictable events can ruin production. We assume that the landlord’s farming endeavours may yield an output $Y_t$ with exogenous probability $P_f$ and zero with probability $1 - P_f$. Similarly, landless sharecroppers may also see their efforts yield no output with probability $1 - P_s$, and $\beta$ with probability $P_s$.

Moreover, an adult faces the likelihood of being unemployed if the demand for labour is less than the population of adults. In every period, for a given level of employment $E$, the probability that any adult will be employed is given by:

$$q_t = \frac{L_t + S_{t+1}}{N}$$

In the face of such uncertainty, each agent will maximize his expected utility conditional on the information set denoted $\Omega_t$.

A risk-averse adult is willing to purchase unemployment insurance against the risk that he is unemployed, while a risk-averse sharecropper will seek insurance against crop failure. Access to contingent-claims contracts, which would efficiently price these risks are limited because capital markets are incomplete. When insurance services are granted through the family, the insurance provider can enforce the terms of the contract efficiently; the elder verifies that the adult participates in the labour market, and conversely, the adult
makes sure that the elder does not shirk in his sharecropping activities.

Assume that contingent claims contracts are signed exclusively between a specific elder and a specific adult such that there is no competition in the provision of insurance services. Typically one does not observe families competing against each other in providing such services to other households’ members for moral hazard and adverse selection reasons. The elder in the family has better information about the types of his descendants and reciprocally. In particular, an elder who is sufficiently less risk-averse than his adult may be willing to provide unemployment insurance. On the other hand, a sufficiently risk-averse elder may purchase crop failure insurance from his less risk-averse adult. Consequently, it is the sign of the difference between the two individual’s risk aversion which matters rather than the magnitude of these differences. Hence, we normalize the preference parameter, $\gamma_i$ of individual $i$ with the smaller risk aversion to 1, such that he becomes risk-neutral.

The insurance provider bears a state-independent cost, $e \neq 0$ in supplying insurance services. When $e < 0$, this cost can be viewed as representing enforcement cost such that the insurer must make sure that the insuree either participates on the labour market or does not shirk when he is a sharecropper. However, the insurer can benefit from family goods, which are by nature similar to public goods, provided by the insuree such that we do not restrict $e$ to be negative. Therefore, $e > 0$ would correspond to a positive effect of the family experienced by the insurer. The insurer’s utility, who is risk neutral, is given by:

$$U(c; 1) + e = c + e \quad (2.5)$$

If insurance contracts are feasible, families will be observed provided such contracts are Pareto-optimal for both family members.
To solve for the optimal level of employment and determine the conditions under which families exist in our village economy, we assume the following timing of events:

1. At the beginning of every period $t$, the landlord announces his demand for unskilled labour $q_t$.

2. At the beginning of period $t$ the landlord offers sharecropping contracts to $S_{t-1}$ of the labourers who worked for him in $t - 1$.

3. Each elder’s status is revealed: receives or does not receive a sharecropping offer.

4. Conditional on the information set $\Omega_t = \{ \text{elders’ status}, q_t, P, \gamma_a, \gamma_o \}$:
   
   (a) Each adult decides whether to purchase UI services from the specific elder or to provide crop failure insurance to that elder.

   (b) Each elder decides whether to provide UI services to the specific adult or to purchase crop failure insurance from that elder.

   (c) The insurance market is revealed.

5. The elder accepts or rejects the sharecropping offer.

6. The landlord randomly hires labourers.

7. At the end of period $t$ the landlord and sharecroppers, if any, observe whether their farming endeavours succeed or fail.

We construct an intra-family redistributive scheme such that the risk-averse individual is indifferent between accepting or rejecting the family insurance contract, and the risk-neutral agent anticipates positive net family transfers through the insurance services he provides.
3 Contracts

Define the following two sets $A = \{\text{employed, unemployed}\}$ and $O = \{\text{unemployed, successful sharecropper, failed sharecropper}\}$ as the states of nature relevant for an adult and an elder respectively. Hence, for a given representative pair of adult and sharecropper, nature has six possible outcomes represented by the pairs $(a, o)$, for $a \in A$ and $o \in O$, and there arise possibilities for risk-sharing associations. We can formally define two contingent-claims contracts observed in our economy.

**Definition 1 (Family)** A family is an insurance contract between a specific elder and a specific adult, which stipulates:

1. State-contingent premiums, $\Phi : A \times O \rightarrow \mathbb{R}_+$, paid by the insuree to the insurer.

2. A state-independent benefit $\psi$ accruing to the insuree.

In our setting, as there is no competition in the insurance market, the insurer is likely to reap positive anticipated profits. Such insurance contracts are not in general actuarially fair. This prompts the risk-neutral family member to insure the risk-averse member. Using Definition 1, it will be possible to determine the conditions under which such insurance contracts are signed and hence discuss the conditions under which families exist. However, not all insurance contracts are feasible.

**Definition 2 (Feasible Intra-Family Transfers)** An insurance contract is feasible if and only if:

1. The insuree pays a premium only in the good state.

2. State-independent benefits paid to the insuree plus enforcement costs never exceed the insurer’s revenue in each relevant state.

The first condition in Definition 2 rules out the case where an unemployed adult would have to pay a premium. Typically in this case the agent has no income and is unable to pay a premium in the ‘bad’ state. As insurance premium is paid only in the case of success, and benefits received in case of failure, moral
hazard problems may arise. An insured adult may not participate in the labour market and a sharecropper may shirk. However, we assume that the family is an efficient institution which prevents such behavior with minimum surveillance and enforcement costs. The last condition in Definition 2 guarantees that the insurer has an income level which allows him to support the insuree when the latter fails. In particular, it rules out the presence of families in the event that the elder is unemployed, i.e. has not received a sharecropping offer. In this case, clearly the elder does not demand crop failure insurance, and the UI plan is not feasible when the adult is unemployed as well. Therefore, the relevant state space is obtained by re-defining $O = \{\text{successful sharecropper, failed sharecropper}\}$.

**Definition 3 (Optimal Intra-Family Transfers)** An insurance contract is optimal if both the insuree and the insurer are at least as well-off living in the family as they are living separately.

The third definition makes use of the rule that two parties to a contract will sign it if and only if it is Pareto-improving. To investigate the existence of families when the elder works for the landlord we first specify the terms of the contract which bind the landlord and the elder.

**Definition 4 (Sharecropping)** A sharecropping contract stipulates the state-contingent rent $\Lambda : A \times O \rightarrow R$, which the sharecropper pays to the landlord.

At any given moment in time the landlord can face unattached individuals or families. In the last case, depending on the adult’s and elder’s risk aversion, the insurer within the family will be different. As a risk-neutral agent with no asymmetry of information and a monopsonist on the sharecropping market, the landlord will choose the state-contingent rents $\Lambda(a, o)$ such as to extract all the sharecropper’s rent in every state of nature. These shares are given in the following Theorem.

**Theorem 1 (Sharecropping contracts)** The landlord’s expected rent from each sharecropper for:

1. A risk-averse elder and risk-neutral adult is:

$$E[\Lambda(a, o)] = \max \{P_s \beta + eW, P_a \beta\}. \quad (3.1)$$
2. A risk-neutral elder and a risk-averse adult is:

\[ E[\Lambda(a,o)] = \max \left[ P_s \beta + (q_t - q^*_t + e)W, P_s \beta \right], \]

(3.2)

where \( e \) is renormalized to \( eW \), \( q_t \) is the probability that the adult is employed, and \( E \) is the unconditional expectations operator.

Proof. See Appendix.

Theorem 1 derives from the principle that the landlord chooses the rent \( \Lambda(a,o) \) such that the insurer is willing to provide insurance services. Moreover, these rents must be such that the conditions enumerated in Definitions 3 and 4 are met. When the elder faces unattached individuals, he chooses those rents such that the sharecropper is indifferent between accepting the contract and being unemployed. In the latter case the elder would earn nothing. As a result the lower bound on the expected rent received from each sharecropper is \( P_s \beta \), that is each sharecropper’s expected output. When the elder provides UI services to an adult, he derives expected monopoly insurance profits which adds up to his expected sharecropping activities and modifies the landlord’s expected rent as given in equation (3.2). The landlord who is a monopsonist extracts all this rent from the sharecropper. The same intuition applies for (3.1) when the adult insures the elder.

Clearly from the three scenarios enumerated in Theorem 1 the landlord will prefer that structure which gives him the highest rent per sharecropper. We now proceed to investigate the conditions under which families exist and its impact on the landlord’s profits.

Theorem 2 (Family) For an insuree’s preferences (2.3), an insurer’s preferences (2.5), and the landlord’s expected output shares defined in Theorem 1:

1. Families always exist for \( e \geq 0 \).

2. For \( e < 0 \) families exist only in the elder-insurer case, and if

\[ e > \gamma \hat{\gamma}(1 - \gamma), \]

(3.3)
then there exists fixed probabilities $q(\gamma, e)$, and $\tilde{q}(\gamma, e)$, with $0 < q(\gamma, e) < \tilde{q}(\gamma, e) < 1$, such that families exist for $q_t \in [q, \tilde{q}]$. Families do not exist otherwise.

Proof. See Appendix.

The intuition behind Theorem 2 is illustrated in Figure 1, which plots an elder’s expected net insurance revenue for two values of the preference parameters: $\gamma = 2.0$ (top curve) and $\gamma = 1.5$ (bottom curve). The family factor $e$ equals $-0.10$ in both cases. The expected UI revenue function is continuous and concave for finite $\gamma$, and reaches a unique maximum at $\hat{\gamma} \equiv \gamma^{1/(1-\gamma)}$. When the employment probability is low ($q_t < q$), the expected insurance premium is low, and the elder refuses to provide unemployment insurance services. Similarly, when the risk of unemployment is low ($q_t > \tilde{q}$), the expected insurance premium is insufficient to cover enforcement expenses, and the elder does not insure the adult. When the adult’s risk aversion increases (higher $\gamma$), the admissible range for families to exist, $[q, \ldots, \tilde{q}]$, increases. Hence, the probability of observing families increases with the adult’s degree of abhorrence toward risk. In the limit where $\gamma \to \infty$, corresponding to log utility, this region would be $1 + e$, and is only determined by enforcement costs $e < 0$.

Finally as one would expect, $[q, \ldots, \tilde{q}]$ shrinks as $e$ falls, that is enforcement cost increase.

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$^9 q = 0.0988$ for $\gamma = 2$ and $q = 0.1702$ for $\gamma = 1.5$.

$^10 \tilde{q} = 0.8876$ for $\gamma = 2$ and $\tilde{q} = 0.7510$ for $\gamma = 1.5$. 

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4 Labor Allocation

Given output shares $\Lambda(a, o)$ obtained in Theorem 1, the infinitely-lived landlord maximizes the present value of profits. In every period $t$, he chooses $S_{t+1}$ and $L_t$, conditional on $S_t$. Note that the landlord’s choices determine the existence of families when $q_t$, $\gamma$ and $e$ satisfy the condition stated in Theorem 2.

Normalize $\beta = R = N = 1$, such that $S_t, L_t$ and $S_{t+1}$ are in percentages. The landlord’s expected instantaneous profits at $t$, $\pi$, is the sum of the expected profits he derives from his own activities and expected output he receives from sharecroppers:

$$\pi(L_t, S_{t+1}; S_t) = P_t F(q_t, r_t) - W q_t + S_t E[\Lambda(a, o)]$$  \hspace{1cm} (4.1)

where $E$ is the unconditional expectations operator, $F$ is a quasi-concave farming technology and $r_t$ denotes the plot of land farmed by the landlord. Hence the landlord’s problem is:

$$\max_{\{L_t, S_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} (1 + \delta)^{-1} \pi(L_t, S_{t+1}; S_t) \right\}$$  \hspace{1cm} (4.2)

subject to:

$$E[\Lambda(a, o)] = \begin{cases} 
\max[P_s + eW, P_s], & \text{adult-insurer;} \\
\max[P_s + (q_t - q_t^e) + eW, P_s] & \text{elder-insurer}
\end{cases}$$

$$q_t = L_t + S_{t+1}$$

$$L_t + S_{t+1} \leq 1$$

$$r_t + S_t = 1$$

$$S_{t+1} \geq 0; \ L_t \geq 0$$
where $\delta$ is the subjective discount rate. The corresponding Bellman equation for problem (4.2) is:

$$V(S_t) = \max_{\{L_t, S_{t+1}\}} \left\{ P_t F(q_t, 1 - S_t) - W q_t + S_t E[\Lambda(a, o)] + (1 + \delta)^{-1} V(S_{t+1}) \right\}$$  \hspace{1cm} (4.3)

where $V$ is the value function.

Assume interior solutions and consider the elder-insurer case. The Euler equations derived from the first-order and Benveniste-Scheinkman conditions are:

$$P_t F_{1t} - W + (1 - r_t)(1 - \gamma q_t^{\gamma-1})W = 0$$  \hspace{1cm} (4.4)

$$P_t F_{2t} - (q_t - \bar{q}_{t} + \epsilon)W - P_s = 0,$$  \hspace{1cm} (4.5)

where $F_{it}$ denotes the first derivative of farming production $F$ with respect to its $i^{th}$ argument. The adult-insurer case is obtained by imposing $\epsilon > 0$ and setting $\gamma = 1$, while the no-family case corresponds to $\gamma = 1$ and $\epsilon = 0$ in equations (4.4) and (4.5).

The first-order condition for the optimal level of apprentices $S_{t+1}$ (4.5) equates marginal product of land to the expected marginal revenue obtained from an additional sharecropper. Contrary to the case where the landlord faces individuals, marginal revenue is augmented by the insurance revenues that are extracted from the insurer $(q_t - \bar{q}_{t} + \epsilon)W$. By Definition 3, these revenues are always positive, such that (4.5) implies that for a given level of employment $q_t$, land allocated to farming, $(r_t)$ when families are observed is always less than the farming acreage under unattached individuals.

With respect to additional laborers $L_t$, (4.4) equates the marginal product of labour augmented, by the marginal insurance revenues, to the fixed subsistence wage. As discussed in Section 3, when the adult is insured, marginal expected UI revenues are increasing initially, then decreasing after $\tilde{q} \equiv \gamma^{1/(1-\gamma)}$. Therefore, if $q_t \leq \tilde{q}$, the marginal cost of labour is reduced by the increase in UI revenues. In this case, conditional upon a given $r_t$, employment is higher when the landlord faces families. When $q_t > \tilde{q}$, decreasing UI revenues implies that the landlord demands less employment when facing families.
For a specific production function, one could combine these two first-order conditions to yield a first-order non-linear difference equation in $S_t$ and $S_{t+1}$. However, analytical closed-form solutions cannot be obtained. We first assume that there exists a unique steady-state equilibrium $(S_*, L_*)$ and investigate how this equilibrium is modified when we slightly perturb the environment. In the next section we shall numerically compute the steady-state equilibrium for a specific production function.

**Proposition 1 (Labor Allocation)** For a quasi-concave instantaneous profit function (4.1) and unique steady-state equilibrium $q_* = L_* + S_*, r_* = 1 - S_*$,

1. $S_*$ is always increasing in $e$.

2. If

$$P_f F_{12}(q_*, r_*) > W(1 - \gamma q_*^{\gamma-1})$$

(a) $L_*$ is decreasing in $e$ and $W$;

(b) $S_*$ is increasing in $W$;

(c) If, in addition, $q_*^2 \geq 0.37$, then $S_*$ is increasing in $\gamma$, while $L_*$ is decreasing in $\gamma$.

**Proof** See Appendix.

The interpretation of Proposition 1 is as follows. An increase in $e$, (i.e. a decrease in enforcement costs) leads to an increase in expected UI revenues in (4.5), while leaving unaffected (4.4). For (4.5) to hold, the landlord must decrease the surface of land he farms, $r_*$, i.e. $S_*$ increases. Next, condition (4.6) insures that the off-diagonal term of the Hessian of the profit function (4.2) is positive when the landlord faces families. This condition is verified unambiguously for the case of individuals because the right-hand side of condition (4.6) equals zero while the left-hand side is always positive for usual production technologies. In this case, it follows that the demand functions $q_*$ and $r_*$, when the elder is the insurer, have the usual desirable properties. In particular, it is straightforward to show that total employment $q_*$ is increasing in farming probability of success $P_f$ and decreasing in sharecropper’s probability of success $P_s$, while the number of sharecroppers is increasing in $P_s$.
Next, when $e$ increases, since $r_*$ decreases and (4.6) is verified, the marginal contribution of employment to profits falls. Under quasi-concavity of the instantaneous profit function (4.1), the landlord reduces employment for the Euler equation (4.4) to be met. An increase in $W$ always reduces the marginal contribution to profits of employment and farming land. This occurs because both expected UI revenues and expected marginal UI revenues do not increase as rapidly as labour costs. We therefore recover the same comparative statics result as the individuals’ case. Finally, note that $q^*$ is the certainty equivalent of the adult, in percentage of the wage level. If the adult’s degree of risk aversion is low, then an increase in $\gamma$ produces a large increment in expected UI revenues. In consequence, it becomes more attractive for the landlord to hire additional sharecroppers. Given that condition (4.6) is met, this is accompanied by a fall in employment.

To summarize we have derived three conditions under which $r_* = 1 - S_*$ decreases. Since $q_* = L_* + S_*$ also decreases, it must be the case that the steady-state number of laborers $L_*$ has fallen. The limits of our analysis in this section are that we abstracted from actual derivation of steady-state equilibria, and that we assumed that family status was fixed. In what follows, we relax both assumptions and numerically compute steady-state equilibria, while allowing for the endogenous formation of families.
5 Numerical Example

We focus on the elder-insurer example because endogenous switching in family structure is only possible in that case. To solve the landlord’s problem, we resort to numerical methods. From a slightly modified version of the technology advocated by Eswaran and Kotwal (1985) consider the following production function:

\[
F_t = \min[(L_t + S_{t+1}), (1 - S_t)^{1/\phi}],
\]

(5.1)

where \( \phi \geq 1 \) is a fixed parameter. A direct consequence of equation (5.1) is that in every period \( t \), employment is determined by the number of sharecroppers \( S_t \):

\[
L_t + S_{t+1} = (1 - S_t)^{1/\phi} = F_t.
\]

(5.2)

Technology (5.1) implies that, in equilibrium, when land under cultivation is increased by 1%, labour demand must be increased by \( 1/\phi \leq 1 \% \) i.e. the expansion path is concave in \( (r_t, q_t) \) space.\(^{11}\)

Substituting (5.2) into Bellman’s equation (4.3) yields:

\[
V(S_t) = \max_{[L_t, S_{t+1}]} \left\{ (P_t - W)q_t + (1 - q_t^\phi)E[\Lambda(a, o)] + (1 + \delta)^{-1}V(S_{t+1}) \right\}
\]

(5.3)

with \( E[\Lambda(a, o)] = \max[P_s + (q_t - q_t^\phi + e)W, P_s] \) is obtained from Theorem 1 for the elder-insurer’s case. The profit schedule (5.3) is piecewise continuous and displays a switching property. From Theorems 1 and 2, the landlord can obtain at least \( P_s \) from each sharecropper. In addition, he can extract all expected UI revenues from the elder-insurer, which by definition are non-negative for families to exist. As can be seen in Figure 1, these conditions are met for the middle range in adult employment probability \( q_t \), when his level of risk aversion is insufficiently high, and enforcement costs are present \( e < 0 \), in which case \( q_t - q_t^\phi + e \) is positive. From the Leontief technology and the identical marginal productivities assumptions, this problem simplifies

\(^{11}\)This restriction guarantees the quasi-concavity of the value function. It could be related to learning-by-doing effects when the landlord increases the scale of operations.
to a static optimization schedule in total employment $q_t$. Consequently, if one or more steady-state(s) equilibrium exist, the economy jumps immediately to it, starting from any initial condition.

As a benchmark, consider the individuals’ case obtained by imposing $\gamma = 1$ and $e = 0$ in (5.3). Here, the first-order condition is linear function, and the optimal number of sharecroppers $S_{t*}$ is explicitly given by:

$$S_{t*} = S_{*} = 1 - \left[ \frac{(P_f - W)}{\phi P_s} \right]^\frac{1}{1-\phi}$$

(5.4)

The reader can verify that (5.4) satisfies all the intuitive properties as far as an inverse demand function is concerned. In particular, $S_{*}$ is decreasing in the landlord’s probability of success, but increasing in the price of labour, and in the expected sharecropper’s output. Such a closed-form solution as (5.4) cannot be computed in the general case where families can exist. Hence, when families are unstable, the objective function displays kinks at points where families become profitable, i.e. between $q(\gamma, e)$ and $\bar{q}(\gamma, e)$ defined in Theorem 2.

Figure 2 plots an example in which two steady-state equilibria are obtained for arbitrary values of the parameters. In this example, when $q < 0.501$, the expected net UI revenues are small given the large enforcement cost ($e = -0.5$). Therefore, families would have to be subsidized to exist. Since the landlord does not benefit in doing so, elders and adults separate. However, for intermediate employment values $q \in [0.501, 0.921]$, families are profitable and the landlord computes the state-contingent tenancy rates $\Lambda(a, o)$ to extract all net UI revenues. These UI revenues fall rapidly however when the probability of unemployment falls. As a result, the enforcement cost outweighs any expected insurance revenues and families separate again when $q > \bar{q}$.

With respect to steady-state solutions, observe that in our example, both an interior solution $q_* = 0.785$, $S_* = 0.225$ and $L_* = 0.56$, and a corner solution $q_{**} = L_{**} = 1$, and $S_{**} = 0$ solve the optimization problem. Hence, both a positive unemployment / sharecroppers and full employment / no sharecroppers allocation can be observed when the elders are acting as providers of UI services to the adult. Conversely, in the adult-insurer or no insurer cases, unique solutions $q_{**} = L_{**} = 1$, and $S_{**} = 0$ are obtained.
6 Conclusion

This paper has investigated the impact of family organization on the sectoral allocation of labour in agrarian economies where formal insurance markets are absent and the land distribution is skewed. We considered a village economy populated by a unique landlord, landless adults and elders who hedge either unemployment or technological risks through the family. A family is made up of a specific adult and elder such that the insurance market is non-competitive. Moreover, the landlord is a monopsonist and extracts as much rent as possible from the landless agents.

We showed that families always exist when the insurer derives positive benefits, in addition to the insurance premium, from supplying insurance services. When there are costs to providing insurance services, families never exist when the adult is risk-neutral and the elder is risk-averse. However, even when there are costs to providing insurance services, a risk-neutral elder and a risk-averse adult may form a family provided the insurance premium is sufficiently high. This is ruled out when the probability that an adult will be employed is either too low or too high. Hence, in that case, employment levels and the cost of providing insurance lead to the endogenous formation (destruction) of families.

Assuming that the solution to the infinitely-lived landlord’s problem is unique, we proved that an increase in the cost of providing insurance services reduces the number of sharecroppers. Moreover, when the instantaneous profits function is well-behaved, an increase in the wage rate or in the insuree’s risk aversion increases the number of sharecroppers but reduces total employment and the number of laborers. These effects are channeled through higher insurance premiums, which in turn induce the landlord to hire more sharecroppers. Lastly, we provided a numerical example where there exists two steady-state equilibria that solve the landlord’s problem: one with no sharecroppers and full employment and one with sharecroppers and positive unemployment. To the extent that skilled employment is considered as a powerful engine of economic development in the endogenous growth literature, the implications of families are therefore important and go beyond simple inter-generational transfers issues.
Our results imply that policies derived from models which do not explicitly consider families could be misleading. For example, policies designed to implement public unemployment insurance will alter an adult’s reservation income and modify intra-family transfers. Presumably, when access to competitive markets that can price risks at more favorable terms to the insuree is allowed, or when state-sponsored social security schemes are put in place, families would disintegrate in the absence of other – e.g. altruistic – ties between household members. Similarly, population control affects the expected unemployment insurance premium and modifies the relative price of sharecroppers and laborers.

We realize that our results hinge on a number of simplifying assumptions including non-storability of the consumption good, identical contemporary marginal products of skilled and unskilled labour and fixed subsistence wage. We also assumed that families form exclusively for financial reasons, abstracting completely from affective considerations. Future research will be needed to determine the respective influence of each of these assumptions on our results. However we feel that our results are sufficiently encouraging as to warrant further study along these lines.
A Proof of Theorem 1

The proof is organized as follows. We first consider the benchmark no-families case with no insurance contract between landless agents. We compute the landlord’s state-contingent claims in each state. Next we consider the family cases (elder-insurer and adult-insurer scenarios respectively). We compute the certainty equivalent $\psi^i$, $i = a, o$ necessary to entice the riskaverse insuree to accept the insurance contract. Conditional on that $\psi^i$, the landlord chooses $\Lambda(a, o)$ such that the sharecropper accepts the contract and the conditions enumerated in Definitions 2 and 3 are not violated. Note that the landlord needs only consider four relevant states of nature because by definition an unemployed elder is not a sharecropper. For notational simplification we suppress the time subscript in $q_t$.

A.1 No Families

Here $\Lambda(a, o)$ is independent of $a$ and varies only with $o$. Table 1 summarizes the three agents’ income in each state of nature. The landlord chooses $\Lambda(a, o)$ such that the elder is indifferent between accepting or rejecting the sharecropping contract in each state. In the no-families case the elder’s reservation income is zero.

<table>
<thead>
<tr>
<th>Nature</th>
<th>Probability</th>
<th>Adult</th>
<th>Elder</th>
<th>$\Lambda(a, o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(employed, success)</td>
<td>$q \hat{P}_s$</td>
<td>$W$</td>
<td>$\beta - \Lambda(a, o)$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>(employed, failure)</td>
<td>$q(1 - \hat{P}_s)$</td>
<td>$W$</td>
<td>$-\Lambda(a, o)$</td>
<td>0</td>
</tr>
<tr>
<td>(unemployed, success)</td>
<td>$(1 - q)\hat{P}_s$</td>
<td>0</td>
<td>$\beta - \Lambda(a, o)$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>(unemployed, failure)</td>
<td>$(1 - q)(1 - \hat{P}_s)$</td>
<td>0</td>
<td>$-\Lambda(a, o)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Consequently, feasible contracts are such that the landlord extracts $\beta$ when the elder’s sharecropping project succeeds, and nothing when the sharecropper fails, with the result that in each state, the sharecropper receives zero, i.e. his reservation income. The landlord’s expected output per sharecropper is obtained directly as:

$$E[\Lambda(a, o)] = p_s\beta.$$  \hfill (A.1)

when there are no families.

A.2 Elder-Insurer

An adult accepts the insurance contract only if he is as well-off by accepting. Hence, incentive-compatibility requires that:

$$ (\psi^a)^\frac{1}{2} = q W^\frac{1}{2} + (1 - q) 0$$  \hfill (A.2)

where $\psi^a$ is the adult’s certainty equivalent. Equation (A.2) implies that $\psi^a = q^t W$. Conditional on $\psi^a$, the landlord chooses $\Lambda(a, o)$ such that the sharecropping contract is accepted. Table 2 reports the adult’s and the elder’s net revenues, as well as the UI premium $\Phi(a, o)$ and the state-contingent rent accruing to the landlord $\Lambda(a, o)$, for every possible state. As the elder’s reservation income is zero in any state and he is risk-neutral, the landlord chooses $\Lambda(a, o)$ such that it equals the elder’s gross income. Given the probability of success for each agent in each state, we can compute the landlord’s expected income per sharecropper as:

$$E[\Lambda(a, o)] = p_s\beta + (q - q_t^i + e)$$  \hfill (A.3)

when he faces a family where the elder is the insurer.
Table 2: Adult’s and Elder’s Net Revenues, Insurance Premium and Landlord’s Rent: Elder-Insurer.

<table>
<thead>
<tr>
<th>Nature</th>
<th>Adult</th>
<th>Elder</th>
<th>( \Phi(a,o) )</th>
<th>( \Lambda(a,o) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(employed, success)</td>
<td>( \psi^a ) ( - \Lambda(a,o) + \Phi(a,o) - \psi^a + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( W )</td>
<td>( \beta + W - \psi^a + eW )</td>
</tr>
<tr>
<td>(unemployed, success)</td>
<td>( \psi^a ) ( - \Lambda(a,o) + \Phi(a,o) - \psi^a + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( 0 )</td>
<td>( \beta - \psi^a + eW )</td>
</tr>
<tr>
<td>(employed, failure)</td>
<td>( \psi^a ) ( - \Lambda(a,o) + \Phi(a,o) - \psi^a + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( W )</td>
<td>( W - \psi^a + eW )</td>
</tr>
<tr>
<td>(unemployed, failure)</td>
<td>( \psi^a ) ( - \Lambda(a,o) + \Phi(a,o) - \psi^a + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( 0 )</td>
<td>( -\psi^a + eW )</td>
</tr>
</tbody>
</table>

A.3 Adult-Insurer

The reservation income of a risk-averse elder who requests insurance from the specific adult is zero. Consequently an elder will accept an insurance contract where the adult guarantees him a certainty equivalent \( \psi^o = 0 \). Next the landlord chooses output shares such that the conditions stated in Definitions 2 and 3 are respected, and the adult is better off providing insurance to the elder. Bearing in mind that the adult’s reservation income equals \( W \) when he works and zero otherwise, Table 3 gives the net revenues of the adult and the elder as well as the crop failure insurance premium and the landlord’s rent. Using the relevant probabilities that each state of nature is realized, it is straightforward to compute the landlord’s expected income as:

\[
E[\Lambda(a,o)] = P_s \beta + eW \quad \text{(A.4)}
\]

when he faces families where the adult is the insurer. Regrouping equations (A.1), (A.3) and (A.4) yields the conditions given in Theorem 1.

Table 3: Adult’s and Elder’s Net Revenues, Insurance Premium and Landlord’s Rent: Adult-Insurer.

<table>
<thead>
<tr>
<th>Nature</th>
<th>Adult</th>
<th>Elder</th>
<th>( \Phi(a,o) )</th>
<th>( \Lambda(a,o) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(employed, success)</td>
<td>( W + \Phi(a,o) - \psi^o + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( \beta + eW )</td>
<td></td>
</tr>
<tr>
<td>(unemployed, success)</td>
<td>( \Phi(a,o) - \psi^o + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( \beta + eW )</td>
<td></td>
</tr>
<tr>
<td>(employed, failure)</td>
<td>( W + \Phi(a,o) - \psi^o + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( eW )</td>
<td></td>
</tr>
<tr>
<td>(unemployed, failure)</td>
<td>( \Phi(a,o) - \psi^o + eW )</td>
<td>( \psi^o ) ( \beta - \Lambda(a,o) )</td>
<td>( eW )</td>
<td></td>
</tr>
</tbody>
</table>
B Proof of Theorem 2

The landlord deals with families if and only if he benefits from their existence. From equations (3.1) and (3.2) in Theorem 1, this depends on the values taken by $e$ and $q$.

1. For $e \geq 0$, $E[\Lambda(a,o)]$ is larger in the presence of families. This result is easily derived from equation (3.1) in the adult-insurer case. For the elder-insurer, the existence of families depends on the sign of $p(q;\gamma,e) \equiv q - q^\gamma + e$. As $0 \leq q \leq 1$, $\gamma \geq 1$, then $q \geq q^\gamma$, such that $q - q^\gamma \geq 0$. Hence for $e \geq 0$, equation (3.2) implies that families also exist in the elder-insurer case.

2. For $e < 0$, families never exist in the adult-insurer case as $P_s \beta + e W < P_s \beta$. In the elder-insurer case, as $q - q^\gamma \geq 0$ and is bounded over its domain, there exists a lower bound $\hat{e}$, such that if $e > \hat{e}$, then $p(q;\gamma,e) > 0$, when evaluated at its maximum. To derive $\hat{e}$, note that $p(\cdot;\gamma,e)$ is uniformly concave since $\gamma > 1$, and attains a unique maximum at:

$$\hat{q} \equiv \gamma^{\frac{1}{1-\gamma}} = \arg\max_{\{q\} \in (0,1]} p(q;\gamma,e).$$  \hspace{1cm} (B.1)

Next, $p(\hat{q};\gamma,e)$ is unambiguously positive if:

$$\gamma^{\frac{1}{1-\gamma}} - \gamma^{\frac{1}{\gamma}} > -e,$$

which simplifies to:

$$\hat{e} \equiv \gamma^{\frac{1}{1-\gamma}} (1 - \gamma) < e$$  \hspace{1cm} (B.2)

i.e. condition (3.3) in Theorem 2.

Moreover, net expected UI revenues $p(q;\gamma,e)$ are characterized by $p(0;\gamma,e) = p(1;\gamma,e) = e$. By concavity of $p(\cdot;\gamma,e)$, if $e > \hat{e}$, then there exists two roots $0 < q < \hat{q} < 1$, such that:

$$p(q;\gamma,e) \begin{cases} = 0, & \text{if } q = \hat{q} \hat{q}; \\ > 0, & \text{if } q \in (\hat{q},\hat{q}); \\ < 0, & \text{otherwise.} \end{cases}$$  \hspace{1cm} (B.3)

Therefore, families exist only in the intermediate range $q \in [\hat{q},\hat{q}]$. \hfill \blacksquare
C Proof of Proposition 1

We assume there is a unique steady-state equilibrium \( q_* = L_* + S_* \) and \( r_* = 1 - S_* \) around which we perform comparative statics exercises. For a general farming production function, \( F(q, r_t) \), there does not exist closed-form solutions for the steady-state equilibrium. Therefore we differentiate the pair of Euler equations (4.4) and (4.5) with respect to \( q_* \) and \( r_t \) and any parameter of interest, \( \theta \). We evaluate those derivatives at the steady-state equilibrium and then discuss how this equilibrium is modified by using Cramer’s rule.

The first order condition can be written as:

\[
\begin{align*}
\pi_1(q, r_t) &\equiv P_f r_1(q, r_t) - W r_t + (r_t - 1)q_*^{\gamma - 1} W, \\
\pi_2(q, r_t) &\equiv P_f r_2(q, r_t) - (P_s + e W) - (q_* - q_t^*) W.
\end{align*}
\]

(C.1)

(C.2)

Let \( \pi_{ij}(q, r_t) \), \( i = 1, 2; \ j = 1, 2, \) denote the second derivative of the instantaneous profits function with respect to its \( i \)th and \( j \)th argument respectively. As the instantaneous profits function is concave in its arguments, then from Cramer’s rule we have:

\[
\begin{align*}
\text{sign } \left[ \frac{\partial q_*}{\partial \theta} \right] &= \text{sign } \left[ \pi_{12}(q, r_*) \pi_{22}(q, r_*) - \pi_{22}(q, r_*) \pi_{11}(q, r_*) \right] \\
\text{sign } \left[ \frac{\partial r_*}{\partial \theta} \right] &= \text{sign } \left[ \pi_{12}(q, r_*) \pi_{11}(q, r_*) - \pi_{11}(q, r_*) \pi_{22}(q, r_*) \right].
\end{align*}
\]

(C.3)

(C.4)

Given that \( r_* = 1 - S_* \), \( \partial r_*/\partial \theta \) and \( \partial S_*/\partial \theta \) are of opposite signs. Moreover, if \( \partial q_*/\partial \theta \) is negative and \( \partial S_*/\partial \theta \) is positive, it must be the case that \( \partial L_*/\partial \theta \) is negative.

Note that \( \pi_{11}(q, r_*) \) and \( \pi_{22}(q, r_*) \) are negative and

\[
\pi_{12}(q, r_*) = P_f r_{12}(q, r_*) - (1 - \gamma q_*^{\gamma - 1}) W,
\]

(C.5)

is positive if

\[
P_f r_{12}(q, r_*) > (1 - \gamma q_*^{\gamma - 1}) W,
\]

(C.6)

which is condition (4.6) in Proposition 1.

Using equations (C.3), (C.4), (C.5), condition (C.6), and that \( \pi_{11}(q, r_*) \) and \( \pi_{22}(q, r_*) \) are negative, we sign the steady-state equilibrium variation with a change in \( \epsilon, W \) or \( \gamma \).

C.1 Insurance cost \( \epsilon \)

Equation (C.1) implies that \( \pi_{12}(q, r_*) = 0 \), while we conclude that \( \pi_{22}(q, r_*) = -W \) from equation (C.2). Making use of those results in equation (C.4) yields that the sign[\( \partial r_*/\partial \epsilon \)] = sign[\( W \pi_{11}(q, r_*) \)], and the sign[\( \partial q_*/\partial \epsilon \)] = sign[\( -W \pi_{12}(q, r_*) \)]. Hence \( \partial r_*/\partial \epsilon \) is unambiguously negative which implies that \( \partial S_*/\partial \epsilon \) is positive. Similarly, from equation (C.3) we conclude that \( \partial q_*/\partial \epsilon \) is negative when condition (C.6) holds. Consequently \( \partial L_*/\partial \theta \) is also negative.

C.2 Wages \( W \)

Differentiating equations (C.1) and (C.2) with respect to \( W \) yield:

\[
\begin{align*}
\pi_{1w}(q, r_*) &= -r_* + (r_* - 1)q_*^{\gamma - 1} \\
\pi_{2w}(q, r_*) &= -e - (q_* - q_t^*).
\end{align*}
\]

(C.7)

(C.8)

Equation (C.7) is unambiguously negative as \( r_* \) lies between 0 and 1. If the landlord faces families, then from Theorem 1, \( q_* - q_t^* + \epsilon \) must be positive such that (C.8) is negative. Making use of the fact that
both $\pi_{1w}(q_*, r_*)$ and $\pi_{2w}(q_*, r_*)$ are negative in equations (C.3) and (C.4) implies that (C.6) is a sufficient condition for $r_*$ and $q_*$ to be decreasing in $W$, such that $S_*$ is increasing in $W$. Hence $\partial L_*/\partial \theta$ is negative.

### C.3 Preference Parameter $\gamma$

Differentiating the first-order conditions (C.1) and (C.2) with respect to $\gamma$ yield:

\[
\begin{align*}
\pi_{1\gamma} &= (r_* - 1)q_*^2 - 1 W (1 + \gamma \log(q_*)) \\
\pi_{2\gamma} &= q_*^2 W \log(q_*)
\end{align*}
\]

Equation (C.10) is always negative because $q_*$ lies between 0 and 1. Equation (C.9) is negative for $q_*^{\gamma} \geq \exp(-1) = 0.368$. Making use of these results in (C.3) and (C.4) to obtain that condition (C.6) and $q_*^{\gamma} \geq 0.368$ is a sufficient condition for $q_*$ and $r_*$ to be decreasing in $\gamma$, and for $S_*$ to be increasing and $L_*$ decreasing in $\gamma$.

\[
\blacksquare
\]
D Figures

Note: Expected net insurance revenue \( p(q; \gamma, e) \equiv q_t - q_t^\gamma + e \). Top curve: \( \gamma = 2.0, e = -0.10 \); bottom curve: \( \gamma = 1.5, e = -0.10 \).

Figure 1: Expected Net Insurance Revenue
Objective function (5.3), for $\beta = R = N = 1$, $\theta = 0.95$, $P_f = 0.9$, $W = 0.225$, $e = -0.5$, $P_s = 0.6$, and $\gamma = 10$.

Figure 2: OBJECTIVE FUNCTION, LEONTIEF FARMING TECHNOLOGY
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