

# Merger control on two-sided markets: is there need for an efficiency defense?\*

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## Abstract

We study horizontal mergers on two-sided markets between horizontally differentiated platforms. We provide a theoretical analysis of the merger's price effect based on the amount of cost savings it generates, the behavior of outsider platforms, and the degree of two-sidedness. We point out differences as compared with the standard, one-sided merger analysis, and also discuss the merger control policy implications.

Keywords: horizontal merger, two-sided markets, cost savings, merger control

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# 1 Introduction

This is a theoretical paper discussing the price effects of horizontal mergers on two-sided markets. In a four-platform framework allowing for horizontal differentiation, we analyze the final price effect for customers taking into account both the cost savings from the merger and the pricing behavior of outsider platforms. In contrast to standard, one-sided merger theory, the impact of cost savings on prices is not monotonic, and depends on the cross-network externalities. In particular, the market's "two-sidedness" makes the efficiency defense redundant if the merger takes place between distant, less substitutable platforms. However, in the case of merger between neighbor, closely substitutable platforms, the efficiency defense is still necessary if the "two-sidedness" is weak. Our results are thus useful for competition authorities facing the challenge of assessing mergers between competing platforms on two-sided markets.

Starting with Rochet and Tirole (2002, 2003), two-sided markets have been the object of increasing research focus over the past fifteen years, in particular due to the fact that these industries often exhibit business conducts and outcomes that would be sub-optimal on traditional (one-sided) markets. The much quoted example that springs to mind is that of firms, even monopolies, setting prices below cost on one side of the platform so as to maximize overall profit thanks to increasing demand on the other side. In other words, the indirect (or cross-group) network effects at work on two-sided markets possibly lead to outcomes not usually predicted by standard analysis. This holds not only from a positive stand, but also from a normative one, to the extent that the previously-mentioned behavior may simply not represent predatory pricing, and as such should not be subject to antitrust vetoing. And while much has been accomplished regarding unilateral pricing strategies, much is left to be done for the study of coordinated behavior such as pricing following a horizontal merger.

Mergers between rivals typically give rise to enhanced market power and higher prices, thus harming customers. But on two-sided markets another effect may reverse this outcome: the cross-group externalities granting users increased utility from having access to a greater pool of business partners on the other side of the platform may neutralize, and even outweigh the utility loss due to the price increase. As a result, the merger could actually be welfare enhancing rather than welfare detrimental (Evans, 2003). Moreover, it is even possible for post-merger prices to be lower than before merger, due to the fact that the merging platforms internalize the effect of a price increase on the merger partner platform - in other words, the same indirect externality may reverse the typical post-merger incentive to increase prices to exploit market power (see Chandra and Collard-Wexler (2009) and Leonello (2010)). Following this type of argument,

Evans and Schmalensee (2007) remind that traditional merger analysis may still apply when the degree of "two-sidedness" (or the size of network externalities, equivalently) is low enough, whereas Evans and Noel (2008) point out some crucial difficulties raised by using conventional methods to analyze mergers in two-sided markets<sup>1</sup>.

Understanding and correctly predicting the outcome of mergers on two-sided markets is increasingly relevant from the practical, public policy viewpoint (Economides 2008, 2010), given the recent surge in such cases for competition authorities<sup>2</sup>. Interestingly enough, the two-sided nature of the market does not systematically play a role in the decision<sup>3</sup>, and when it does, it plays against the parties: the Norwegian media merger between Edda Media and A-Pressen was cleared in 2012 conditional on structural remedies/asset divestitures on certain local/geographic markets, whereas the Deutsche Börse-NYSE Euronext deal was banned in 2012 due to insufficient cost savings to compensate for the supposedly likely post-merger price increase. Incidentally, such decisions go against the conclusions of most of the literature dealing with the topic, which is mostly empirical, and generally agrees that, at least for the cases studied, horizontal mergers on two-sided market do not lower welfare<sup>4</sup>. Clearly, there is need to further study mergers on two-sided markets, all the more so that few papers address this from the theoretical point of view.

Chandra and Collard-Wexler (2009) specifically study the post-merger pricing in two-sided markets based on a theoretical model: a modified Hotelling duopoly where consumers are assumed to be single-homing, whereas advertisers may advertise in several newspapers. The key finding is that increased concentration may not lead to higher prices on either side, because the resulting monopoly may actually choose to set lower prices. This result is however conditioned on pricing below marginal cost on the reader side: if newspapers sell their content at a price below marginal cost, then additional readers are only valuable to the extent that the revenues which could be made by selling their attention to advertisers are greater than the subsidy. The

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<sup>1</sup>The Lerner index for instance does not hold, and merger price-simulation methods that are now commonly used for traditional markets turn out to be misspecified when applied to multi-sided markets. See Affeldt and al. (2013) for how to adapt the UPP test to two-sided markets.

<sup>2</sup>See for the instance the merger between the Dutch yellow pages directories (Case No. 6246/European Directories, 2008), the Edda Media and A-Pressen merger in Norway (Konkurransetilsynet Case 2011/0925 MAB BMBE) or the Deutsche Börse-NYSE Euronext deal (EC Case No. M.6166-2012).

<sup>3</sup>Incidentally, although a new edition of the US Merger Guidelines was released in 2010, replacing the previous Guidelines from 1992, there was no mention of two-sided markets.

<sup>4</sup>See Chandra and Collard-Wexler (2009) for the Canadian newspaper industry, Filistrucchi et al. (2012) for the Dutch daily newspaper market, and Jeziorski (2014) for the US radio industry.

fact that the data on the Canadian newspaper industry corroborates the absence of price increase following horizontal mergers does not however rule out alternative explanations<sup>5</sup>: it is perfectly possible that some mergers were accompanied by efficiency gains/cost savings, which by virtue of lower costs for the merged parties allowed prices to remain unchanged.

A second theoretical attempt to address optimal pricing after a horizontal mergers on two-sided markets is Leonello (2010). In a merger-to-monopoly scenario, the monopolist will offer advertisers in one newspaper the opportunity to advertise also in the other one as part of the merger deal. For a single price, the advertiser can now reach twice as many consumers as before, which is referred to as 'interoperability'. Leonello (2010) shows that the introduction of advertising bundling by the monopolist increases the incentives to keep prices low on at least one side of the market, because the interoperability increases the margin which the newspaper can charge on advertising, and it thereby becomes profitable to reduce prices on the consumer side in order to stimulate demand. Overall, welfare could increase following a merger, and this result is obtained absent efficiency gains.

We depart from these papers by focusing in contrast on the role played by the merger cost savings for the post-merger pricing strategy of all platforms on the market, both insiders and outsiders, and thereby ultimately for the merger's impact on consumers. This is arguably relevant for competition authorities, since they are bound to explicitly balance the efficiency gains from a merger against its anti-competitive effect in order to decide whether to ban or to clear it<sup>6</sup>. Our formal setting assumes horizontal differentiation of platforms<sup>7</sup> as well as positive and reciprocal valuation between the two sides of the market. As an illustrative example, one can think about competition between stock exchanges, for which each side values the presence of the other, and platform differentiation is triggered by product scope, geography or regulatory constraints<sup>8</sup>. We compute and compare optimal post-merger prices set by both insider and

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<sup>5</sup>It could also be that some mergers were driven by motives other than increased market power, such as for instance empire-building or political motives (Anderson and McLaren, 2012).

<sup>6</sup>See for instance the 2010 US Horizontal Merger Guidelines and the European Commission Merger Regulation 139/2004.

<sup>7</sup>Formally, the paper closest to ours is Brito (2005), except for the two-sided perspective of course. Gal-Or and Dukes (2006) also use the Salop setting to address horizontal media mergers, but they focus exclusively on the role of bargaining power on one side of the market, i.e. between advertisers and radio stations in particular, for the profitability of non-consolidating deals.

<sup>8</sup>Product scope may lead to differentiation because a trader may prefer to trade an asset on the exchange where he already trades other products (see Economides and Siow (1988) for a theoretical treatment). In turn, regulation on cross-border access to exchanges may artificially differentiate traders through different costs of accessing these exchanges (see Cantillon and Yin, 2010, on how membership choices at two exchanges have been accordingly

outsider platforms in two different cases: bilateral merger between adjacent, closely substitutable platforms, or between distant, less substitutable ones. We show that the two-sidedness makes the efficiency defense redundant if the merger takes place between distant platforms. In the case of adjacent merging platforms, however, the efficiency defense is still necessary if the two-sidedness is relatively weak. Furthermore, these conclusions are robust to asymmetric reciprocal valuations between the two sides. Finally, the main takeaways of the paper are that the type of merger is determinant for the scope of the efficiency defense, much as on one-sided markets, and that as long as the two-sidedness is low enough, the same merger control can be safely applied as on one-sided markets.

Before going on to our model, let us note that we perform a significantly more general analysis than what has been previously done in the literature. To start with, we allow for outsiders and no longer consider merger to monopoly, which enables a more thorough analysis since the final price effect depends on the outsiders' pricing strategy, and also makes possible to consider different types of merger, between neighbor, or, on the contrary, distant platforms. Finally, we allow for cost savings, which enables the study of the relationship between merger efficiency gains and indirect network effects from the point of view of optimal post-merger pricing.

The rest of the paper is organized as follows: first we introduce the framework, then the pre-merger equilibrium. The merger analysis begins with an illustrative examples before going on to extend results in a more general case. We discuss the policy implications before concluding. The Appendix at the end of the paper provides the analytical proofs for the illustrative example, whereas the Technical Appendices available upon request provide the detailed Maple computations and Mathematica simulations respectively.<sup>9</sup>

## 2 The model

### 2.1 The framework

Consider a four-platform market. The platforms, denoted  $k \in \{A, B, C, D\}$ , compete in prices and are equidistantly located on the unit circular market. We assume exogenous locations:  $A$  is located in  $0(1)$ ,  $B$  in  $\frac{1}{4}$ ,  $C$  in  $\frac{1}{2}$  and  $D$  in  $\frac{3}{4}$ . Each platform produces or sells the same type of product/service to two groups of customers, located on the two sides of the platform, enabling

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modified). Finally, even within a country, regulation can introduce an artificial differentiation of traders - see Cantillon and Yin (2011).

<sup>9</sup>The Technical Appendices are very space consuming and as such are left out the current version.

them to interact. Let there be  $N_i$  customers uniformly distributed along each side  $i \in \{1, 2\}$  of the market. We assume single-homing throughout the paper: each customer has access to the other side of the market through one platform only<sup>10</sup>. Denote  $v$  the reservation price, the same for all buyers. They all have inelastic demand, buying only one unit of the good/service if the price is less than the reservation price. The platforms do not spatially discriminate their buyers, and so they use mill pricing. Each customer will choose the platform offering the lowest total price, equal to the sum of the transportation cost and the sale price. Note that we assume the platforms to compete in access or membership fees rather than usage fees<sup>11</sup>. We assume linear transportation cost  $t_i d$  on each side  $i$ , where  $t_i$  is the constant unit cost and  $d$  the distance from a platform to the customer's location<sup>12</sup>.

The net utility for a customer on side  $i$  using platform  $k$  writes as follows:

$$U_i^k = v + a_i n_j^k - p_i^k - t_i d_{i,k}$$

where  $p_i^k$  is the price of platform  $k$  on side  $i$ ,  $a_i n_j^k$  the indirect network externality (a customer patronizing platform  $k$  on side  $j$  induces consumer utility equal to  $a_i$  on side  $i$ ), and  $d_{i,k}$  the distance between the customer and the platform  $k$ . For instance, the utility for a customer located on side 1 and joining the platform  $A$  is:

$$U_1^A = v + a_1 N_2 x_2^A - p_1^A - t_1 x_1^A$$

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<sup>10</sup>Horizontal differentiation often results in multi-homing (Evans and Schmalensee, 2007). We restrict however to the single-homing hypothesis for both sides for several reasons. First of all, to preserve tractability. Secondly, we do not allow for multi-homing on only one side either since the customers that multi-home would not take into account the price of other platforms when joining one of them (Durand, 2008), whereas we want to study the consequences of mergers between platforms competing on both sides. Finally, within our illustrative example of competing exchanges, where the listing fee will be the main pricing strategy that we consider on behalf of the platforms, a company's primary listing will be unique, typically a domestic one, although it may also have several international listings - see §38 to 42 of the Deutsche Borse-NYSE Euronext decision/EC Case No. M.6166-2012.

<sup>11</sup>In the context of our illustrative example based on competition between exchange platforms, this means that we focus on membership and/or listing fees instead of the trading and brokerage fees that are incurred on a per-transaction basis.

<sup>12</sup>Arguably, the choice of linear transport cost may seem restrictive, to the extent that the initial spatial pattern that we consider on each side of the market, that of equidistant locations, has only been rationalized as a location equilibrium for the pure-strategy location-then-price game in the case of quadratic transport cost (Economides 1989). However, we do not deal with location choices, and neither do we allow for relocation, and therefore maintain our choice of a linear transport cost for obvious tractability reasons. Furthermore, both the linear and the quadratic transport cost versions of the Salop model yield the same demand and consumer surplus functions (see Hoernig 2015), therefore there is no loss of generality for our side-by-side price analysis given the fixed locations assumption.

where  $x_1^A$  stands for the distance on the side 1 of the market traveled by the customer located in  $x_1^A$  to join the platform  $A$ . Let  $a_i \geq 0, N_i \geq 0$ , and assume the intrinsic utility  $v$  to be high enough to always guarantee full market coverage.

## 2.2 Pre-merger analysis

We set up as a benchmark the pre-merger case in which all the four platforms compete to attract customers located on both sides bearing a same unit production cost  $c$ . Hence, each platform  $k$  will maximize the total profit it makes on both sides of the market by optimally setting prices  $p_i^k$ . In order to determine the pre-merger price equilibrium, one needs first to establish the system of demands, and for this one has to write the demand served by platform  $k$  on each side.

Since platforms are located on a circle, each platform  $k$  has two neighbors, namely the competing platforms  $k_l$ , on its left, and  $k_r$ , on its right. Then, the marginal customer  $x_i^{k,k_l}$  between platform  $k$  and its neighbor  $k_l$  on side  $i$  is defined by

$$v + a_i N_j x_j^k - p_i^k - t_i x_i^{k,k_l} = v + a_i N_j x_j^{k_l} - p_i^{k_l} - t_i \left( \frac{1}{4} - x_i^{k,k_l} \right), \quad (1)$$

while the marginal customer between  $k$  and  $k_r$  on the same side of the market is defined by:

$$v + a_i N_j x_j^k - p_i^k - t_i x_i^{k,k_r} = v + a_i N_j x_j^{k_r} - p_i^{k_r} - t_i \left( \frac{1}{4} - x_i^{k,k_r} \right). \quad (2)$$

The total demand for platform  $k$  on side  $i$  is therefore given by  $x_i^{k,k_l} + x_i^{k,k_r}$ , where

$$x_i^{k,k_l,r} = \frac{4a_i N_j x_j^k - 4p_i^k - 4a_i N_j x_j^{k_l,r} + 4p_i^{k_l,r} + t_i}{16t_i} \text{ where } k_{l,r} = k_l, k_r.$$

Solving the system of total the demands yields the individual demand  $x_i^k$  for each platform  $k$  and for each side  $i$  of the market as functions of all prices. Finally, total profit for platform  $k$  writes as follows:

$$\Pi^k = (p_i^k - c)N_i x_i^k + (p_j^k - c)N_j x_j^k.$$

Each platform  $k$  maximizes its individual profit setting simultaneously its prices  $p_i^k$  and  $p_j^k$  for the two sides. Hence, the vector of equilibrium prices  $\left( (p_i^k)^*, (p_j^k)^* \right)_{k=A,B,C,D}$  solves the system of the FOCs, i.e.

$$\left( p_i^k \right)^* = \arg \max_{p_i^k} \left( N_i x_i^k (p_i^k - c) + N_j x_j^k (p_j^k - c) \right), i, j = 1, 2, k = A, B, C, D.$$

Due to the symmetry between the four platforms on each side of the market, we obtain a symmetric equilibrium  $p_i^* = (p_i^k)^*$  in which platforms set the same price on each side. The equilibrium prices are given by:

$$p_i^* = \frac{2(t_i)^2 t_j - 18ca_j N_i a_i N_j - 3N_j a_j t_i t_j + 8ct_i t_j - 5a_j N_i a_i N_j t_i + 6(N_j)^2 (a_j)^2 N_i a_j}{2(4t_i t_j - 9a_j a_i N_i N_j)}$$

Equilibrium profits are in turn given by  $\Pi^* = (p_i^* - c)N_i x_i^* + (p_j^* - c)N_j x_j^*$ , where  $x_i^*$  is the individual demand on side  $i$  at the symmetric equilibrium.

### 3 Merger analysis

Our purpose is to analyze the post-merger price behavior of both insider and outsider platforms, so as to understand the merger's likely effect on customers. Insiders' post-merger prices will reflect the amount of cost savings obtained through merger, and the outsiders' prices will change in response to the insiders' pricing strategy. By comparing post- and pre-merger equilibrium prices, we will be able to make clear to what extent the 'two-sidedness' of the market changes results as compared with the 'standard' merger analysis performed on one-sided markets.

#### 3.1 Preliminary remarks

In the following, we first remind the traditional results from the "one-side market" merger analysis and then present the simplifying assumptions we are going to make before discussing the illustrative example.

**Traditional/one-sided markets results.** The main insights for the traditional/one-sided market merger analysis can be summarized as follows: without cost savings, a merger between neighbor firms competing in prices on the circular market increases all prices, but the farther away the outsiders, the lower their price increase (Levy and Reitzes, 1992). In turn, the merger between distant firms should not affect prices, nor profits, in the absence of cost savings, simply because for the market-power price-increasing effect to arise, the two merger firms need some captive demand, which is missing given that they are not neighbors. Thus, the spatial literature on horizontal mergers between firms competing in prices on the circular market (Levy and Reitzes (1992), Brito (2003, 2005)) concludes that a bilateral merger between neighbors necessarily involves a unilateral market power effect and as such is always profitable, whereas a merger between non adjacent firms can only be motivated either by cost savings or possibly to better sustain collusion. Let us now also briefly recall the usual post-merger trade-off arising whenever

a merger may lead to cost savings. On the one hand, the market power effect pushes the price upwards, so as to increase profits over the residual captive demand served by the two merging partners. But on the other hand, the cost savings increase their productive efficiency relative to their rivals, enabling the insiders to lower their prices, attract more customers, and thereby increase their profits. The net price effect for the insider firms naturally depends on the amount of cost savings, and in what follows we shall focus on identifying the amount of cost savings that make the merger neutral from the point of view of customers.

**Simplifying assumptions.** For clarity reasons, given the four-platform setting we consider, we use hereafter several simplifying assumptions.

We leave aside the merger's impact on locations, so we only consider exogenous differentiation. In other words, we rule out post-merger product repositioning. We equally neglect the possibility for the merger to trigger changes in the number of active outlets or products, as well as any vertical differentiation issues (see Fan 2013 and Jeziorski 2014 for examples of empirical analyses taking this into account). Also, we assume that although the merger does give access to more users/business partners on the other side of the market, this will not change their willingness to pay or intrinsic utility from joining a platform<sup>13</sup>. In other words, we simply assume that the merger will change the ownership pattern in the industry, and lead to a joint pricing decisions for the merging platforms, but will not alter the type of equilibrium that the platforms play, nor change the users' preferences.

Furthermore, we only consider perfectly symmetric differentiation between the four platforms on both sides of the market, i.e. let  $t_1 = t_2$ , and also assume equal market size for the two sides connected by the platforms, i.e.  $N_1 = N_2$ . We further normalize these two parameters, by setting  $t = 1$  and  $N = 1$ . We therefore restrict to a single source of exogenous asymmetry between the two sides, in the shape of possibly different intensity of the indirect externalities. More precisely, let  $a_1 = a$  but  $a_2 = \beta \times a$ . Parameter  $\beta$  measures the asymmetry between sides in term of valuation of the presence of the other side. For instance, with  $\beta = 0$ , side 2 does not value the presence of side 1, although the latter does value the presence of side 2. Note that we shall only restrict to positive cross-group externalities, i.e.  $a \geq 0$ <sup>14</sup>. On the contrary, for

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<sup>13</sup>In practice, mergers are often consummated to increase customer valuation of the service supplied by the entities in question. We want to focus though on increases in the net utility from joining a platform that are endogenously impacted by the merger, to the extent that it is the change of ownership that provides access to a greater pool of business partners on the other side. For an example of horizontal merger with exogenous change in the base valuation of customers, see Leonello (2010).

<sup>14</sup>This is consistent by the way with our underlying example of competing exchanges. Of course, this positivity

$\beta = 1$ , the cross-group externalities are identical for the two sides of the market, meaning that customers on each side of the market value the other side participation with the same intensity.

Given all the above simplifications, pre-merger prices become

$$p_1^* = p_2^* = p^* = \frac{1}{2} \frac{-6a^3\beta + 5a^2\beta + 18ca^2\beta + 3a - 8c - 2}{9a^2\beta - 4},$$

whereas all pre-merger profits equal

$$\Pi^* = \frac{1}{8} \frac{6a^3\beta^2 + 6a^3\beta - 10a^2\beta - 3a\beta - 3a - 4}{9a^2\beta - 4}.$$

### 3.2 An illustrative example: perfect symmetry between market sides

Here we set up the perfect symmetry case as an illustrative example to give some first insights on how merger may affect prices and profits. All the previous simplifications boil down to two-parameter profit and price functions for the pre-merger equilibrium. Nonetheless, close-form solutions for the SOCs turn out to be out of reach, making necessary Mathematica simulations. Given that the post-merger equilibrium will necessary involve still one more parameter, the cost savings, we begin our merger analysis by a simple but insightful example: we assume in this section perfect symmetry between side, i.e.  $\beta = 1$ , so as to keep a two-parameter framework. Below we provide the results for this simple case, then in the next section go on to extend them by simulating the outcome of a more general framework allowing for asymmetry between market sides.

#### 3.2.1 Merger between adjacent platforms

Suppose that platforms  $A$  and  $B$  merge. As before mentioned, we do not allow for platform relocation, nor changes in the customers' willingness to pay. The merger simply allows customers on one side of the market to reach more customers on the other side, although they still connect *via* a single platform. The merger does not involve shutting down any of the platforms, so post-merger there will still be eight prices to be determined: the joint pricing decision for  $A$  and  $B$  will lead to different values for  $p_1^A, p_1^B, p_2^A$  and  $p_2^B$ , triggering in response different values for  $p_1^C, p_1^D, p_2^C$  and  $p_2^D$ . In fact, the market turns into an asymmetric triopoly, since we allow for cost savings/efficiency gains from merger: the merged platform will operate with a lower constant unit cost as compared with the remaining outsider platforms. In order to keep the analysis 

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constraint also implies that we do not allow for disutility caused by one of the sides to the other, as may happen for instance in some two-sided media models of readers or viewers and advertisers.

tractable, we do not allow for different cost savings between the two sides of the market. Denote  $c - \delta$  the unit cost for the group  $A + B$ , where  $\delta \in [0, c]$ . Then the merged entity will now maximize

$$\begin{aligned}\Pi^A + \Pi^B &= (p_1^A - c + \delta)N \left( x_1^{A(D)} + x_1^{A(B)} \right) + (p_2^A - c + \delta)N \left( x_2^{A(D)} + x_2^{A(B)} \right) \\ &+ (p_1^B - c + \delta)N \left( x_1^{B(C)} + x_1^{B(A)} \right) + (p_2^B - c + \delta)N \left( x_2^{B(C)} + x_2^{B(A)} \right),\end{aligned}$$

whereas the platforms  $C$  and  $D$  go on maximizing their stand-alone profits:

$$\begin{aligned}\Pi^C &= (p_1^C - c)N \left( x_1^{C(D)} + x_1^{C(B)} \right) + (p_2^C - c)N \left( x_2^{C(D)} + x_2^{C(B)} \right), \\ \Pi^D &= (p_1^D - c)N \left( x_1^{D(A)} + x_1^{D(C)} \right) + (p_2^D - c)N \left( x_2^{D(A)} + x_2^{D(C)} \right).\end{aligned}$$

Due to the symmetry between the insider platforms, as well as between the outsiders, and following all the above simplifications, the following optimal post-merger prices obtain for the insider and outsider platforms respectively:

$$\begin{aligned}\tilde{p}^{A+B} &= c + \frac{10a\delta - 6\delta - 11a + 7a^2 + 4}{10 - 16a}, \\ \text{and } \tilde{p}^C &= \tilde{p}^D = c + \frac{4a\delta - 2\delta - 9a + 6a^2 + 3}{10 - 16a}.\end{aligned}$$

Note that:  $\tilde{p}^{A+B} - c + \delta \geq 0, \forall \delta \in [0, c]$ , but  $\tilde{p}^C - c \geq 0$  iff  $\delta \leq \frac{3}{2}(1 - a) \equiv \bar{\delta}$ .

Let  $f(\delta, a)$  stand for the post-/pre-merger price difference for the insider platforms on either side of the market:

$$f(\delta, a) = \tilde{p}^{A+B} - p^* = \frac{(3 - 5a)(6a\delta - 4\delta - 2a + 1 + a^2)}{(4 - 6a)(5 - 8a)}.$$

By the same token, the outsider-platform price difference between merger and no-merger on either side of the market writes

$$h(\delta, a) = \tilde{p}^C - p^* = \frac{1(1 - 2a)(6a\delta - 4\delta - 2a + 1 + a^2)}{2(2 - 3a)(5 - 8a)}.$$

Moreover, profits for the insiders and outsiders write as follows:

$$\begin{aligned}\tilde{\Pi}^{A+B} &= \frac{1}{2} \frac{(4\delta - 11a - 6a\delta + 4 + 7a^2)^2}{(1 - a)(5 - 8a)^2} \text{ and} \\ \tilde{\Pi}^C = \tilde{\Pi}^D &= \frac{1}{4} \frac{(2a - 1)(3a - 2)(3a - 3 + 2\delta)^2}{1 - a(5 - 8a)^2} \equiv \tilde{\Pi}^O \text{ respectively.}\end{aligned}$$

Therefore, the profit differentials for insiders and outsiders are given by:

$$\begin{aligned}\tilde{\Pi}^{A+B} - 2\Pi^* &= \frac{4(2-3a)^3\delta^2 + 4(4-7a)(1-a)(2-3a)^2\delta + (19a^2 - 23a + 7)(1-a)^3}{2(1-a)(8a-5)^2(2-3a)} \\ \text{and } \tilde{\Pi}^O - \Pi^* &= (1-2a)\frac{(4-6a)^2\delta^2 - 12(1-a)(3a-2)^2\delta + (1-a)^3(11-17a)}{4(1-a)(2-3a)(8a-5)^2}.\end{aligned}$$

Based on the monotonicity of price and profit differentials (see the Appendix), and restricting the analysis to  $a \in [0, 0.5) \cup (0.75, 1)$  satisfying the SOCs (see also the Appendix), the following results hold:

**Result 1:** *There exists a unique and strictly positive threshold of cost savings  $\tilde{\delta}$  such that the adjacent merger leaves all prices unchanged.*

It is straightforward to check that  $\tilde{\delta} = \frac{(a-1)^2}{2(2-3a)} > 0$  solves for  $f(\delta, a) = h(\delta, a) = 0$ . This leaves scope for an efficiency defense, as shown in the result below:

**Result 2:** *(i) For a weak indirect externality, i.e.  $a \in [0, 0.5)$ , the adjacent merger leads to lower prices iff the merger generates enough cost savings ( $\delta \in [\tilde{\delta}, \bar{\delta}]$ ); moreover, the merger is always profitable for the insiders.*

*(ii) For a strong indirect externality, i.e.  $a \in (0.75, 1)$ , the adjacent merger always leads to lower prices, regardless of the amount of merger efficiency gains; however, the merger is profitable for the insiders iff it generates enough costs savings ( $\delta \in [\delta^*, \bar{\delta}]$ ).*

*See proof in the Appendix.*

Result 2 provides the merger's impact on consumers' welfare depending on the intensity of the cross-group externality between the two sides of the market. If this externality is weak, post-merger prices fall for high enough cost savings generated by the merger. Instead, the merger's efficiency gains no longer matter when the externality is strong, because in that case post-merger price decreases anyway. In other words, Result 2 points out a certain degree of substitutability between the merger's efficiency gains and the market's two-sidedness: the higher the latter, the lower the importance of cost savings.

To better grasp this result, recall that the cross-group externality favors customers to the extent that it provides incentives for firms to lower their prices so as to better benefit from the increase in demand that it gives rise to - basically, it works similarly to cost savings from the point of view of firms' profits. As a result, a weak indirect externality cannot fully compensate

the market power effect of the merger, and it takes high enough cost savings for the insiders to lower their price. However, as soon as the indirect externality is strong enough, the merger cost savings are less needed for the price to diminish. The final price effect for customers is unambiguous, since the outsider platforms will also lower their prices. Note finally that it is the intensity of the cross-group externality that is essential for the pricing behavior of the merging firms, or, equivalently, Result 2 is indeed driven by the market's two-sidedness. For this, consider the case of zero cost savings: on one-sided markets, post-merger prices necessarily increase, whereas here customers still benefit from the merger provided that the externality is high enough.

Nevertheless, consumers would not be given the opportunity to benefit from the merger if the latter is not submitted in the first place. Note therefore that Result 2 also discusses the merger's profitability, thus making clear the situations where it is indeed rational to look for the efficiency gains justifying the merger's approval<sup>15</sup>. Again, two cases arise. First, when the cross-group externality is weak, the merger between neighbor platforms is always internally profitable, which basically extends the one-sided outcome, since a merger between neighbor firms competing in prices is always profitable for them (see Levy and Reitzes (1992) or Brito (2005)), all the more so when it involves cost savings. Incidentally, this means that the market power effect of the merger (i.e. exploiting its captive demand) is relatively dominant, since the merger is profitable even with zero indirect externality and zero cost savings. Secondly, when the cross-group externality is strong, it takes high enough efficiency gains for the merger to be profitable. This is reminiscent of Cournot mergers, where the market power effect cannot compensate for business stealing<sup>16</sup>, thus making cost savings a *sine qua non* condition for the merger to take place. However, here we only deal with price-setting platforms, and moreover, the business stealing effect is completely reversed to favor customers and hurt outsiders: the latter do not benefit from it even though the cost savings may be too low for the merger to be profitable. This is easily explained by the merger's impact on prices. To begin with, whenever the merging platforms lower their prices, outsider firms will do the same, but less than the insiders, since they do not enjoy cost savings. As a result, they will lose some demand in the process, and therefore the combined final effect

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<sup>15</sup>The computations in the Appendix show that for the range of parameters in Result 2, the outsiders do not exit the market, even though they may be hurt by the merger.

<sup>16</sup>In the Cournot case, the business stealing is due to outsiders' serving an increased residual demand when the insiders push the post-merger price upwards by restricting their joint output. In our setting, the business stealing effect would stand for the fact that the insiders' price drop does not bring about enough demand to increase profits, because the outsider platform respond by also lowering their price.

for their profits is unambiguous: they are harmed by the merger whenever customers benefit from it through lower prices.

As far as the implications for competition policy go, they are straightforward, and on this point our analysis formally confirms the intuition of Evans and Schmalensee (2007) that the 'traditional' merger analysis still applies when the degree of 'two-sidedness' is low enough. According to Result 2, the efficiency defense is necessary only if the indirect externality is weak, because in that case the merger only improves consumers' welfare for high enough cost savings. In turn, a strong indirect externality guarantees that the customers benefit from the merger, but the latter only takes place for high enough cost savings: the efficiency defense is thus redundant.

### 3.2.2 Merger between distant platforms

Let us now consider the alternative type of merger, i.e. the merger between distant platforms A and C. Again, assuming equal cost savings between sides, the merged platform will operate with a lower constant unit cost as compared with the remaining outsider platforms. Denote now  $c - \lambda$  the unit cost for  $A + C$ , where  $\lambda \in [0, c]$ . Then the merged entity will now maximize

$$\begin{aligned} \Pi^A + \Pi^C &= (p_1^A - c + \lambda)N \left( x_1^{A(D)} + x_1^{A(B)} \right) + (p_2^A - c + \lambda)N \left( x_2^{A(D)} + x_2^{A(B)} \right) \\ &\quad + (p_1^C - c + \lambda)N \left( x_1^{C(D)} + x_1^{C(B)} \right) + (p_2^C - c + \lambda)N \left( x_2^{C(D)} + x_2^{C(B)} \right), \end{aligned}$$

whereas the platforms  $B$  and  $D$  go on maximizing their stand-alone profits:

$$\begin{aligned} \Pi^B &= (p_1^B - c)N \left( x_1^{B(C)} + x_1^{B(A)} \right) + (p_2^B - c)N \left( x_2^{B(C)} + x_2^{B(A)} \right), \\ \Pi^D &= (p_1^D - c)N \left( x_1^{D(A)} + x_1^{D(C)} \right) + (p_2^D - c)N \left( x_2^{D(A)} + x_2^{D(C)} \right). \end{aligned}$$

As before, due to the symmetry between the insider platforms, as well as between the outsiders, we obtain the following optimal post-merger prices:

$$\begin{aligned} \widehat{p}^{A+C} &= \frac{24c - 19a - 32ac - 16\lambda + 20a\lambda + 14a^2 + 6}{8(3 - 4a)}, \\ \text{and } \widehat{p}^B &= \widehat{p}^D = \frac{12c - 9a - 16ac - 4\lambda + 4a\lambda + 6a^2 + 3}{4(3 - 4a)}. \end{aligned}$$

Note that:  $\widehat{p}^{A+C} - c + \lambda \geq 0 \forall \lambda \in [0, c]$  but  $\widehat{p}^B - c \geq 0$  iff  $\lambda \leq \frac{3}{2} \left( \frac{t}{2} - z \right) \equiv \bar{\lambda}$ .

Let now  $g(\lambda, a)$  stand for the post/pre-merger price difference for the insider platforms on either side of the market:

$$g(\lambda, a) = \widehat{p}^{A+C} - p^* = \frac{(4 - 5a)(12a\lambda - 8\lambda - a + 2a^2)}{8(3a - 2)(4a - 3)}.$$

By the same token, the outsider-platform post-/pre-merger price difference on either side of the market writes

$$l(\lambda, a) = \hat{p}^B - p^* = \frac{(1-a)(12a\lambda - 8\lambda - a + 2a^2)}{4(3a-2)(4a-3)}.$$

Moreover, post-merger equilibrium profits are now given by:

$$\begin{aligned} \hat{\Pi}^{A+C} &= \frac{\begin{pmatrix} 288\lambda - 780a - 1120a\lambda + 1539a^2 - 1316a^3 + 412a^4 \\ -64\lambda^2 + 192a\lambda^2 + 1448a^2\lambda - 624a^3\lambda - 144a^2\lambda^2 + 144 \end{pmatrix}}{64(1-a)(4a-3)^2} \text{ and} \\ \hat{\Pi}^B = \hat{\Pi}^D &= \frac{(8\lambda - 27a - 12a\lambda + 14a^2 + 12)(20a\lambda - 16\lambda - 19a + 14a^2 + 6)}{64(1-a)(4a-3)^2} \equiv \hat{\Pi}^O, \end{aligned}$$

therefore the profit differentials equal:

$$\begin{aligned} \hat{\Pi}^{A+C} - 2\Pi^* &= \frac{\begin{pmatrix} 72a - 576\lambda + 3104a\lambda - 394a^2 + 785a^3 - 676a^4 + 212a^5 + 128\lambda^2 \\ -576a\lambda^2 - 6256a^2\lambda + 5592a^3\lambda - 1872a^4\lambda + 864a^2\lambda^2 - 432a^3\lambda^2 \end{pmatrix}}{64(1-a)(3a-2)(4a-3)^2} \text{ and} \\ \hat{\Pi}^O - \Pi^* &= \frac{(12a\lambda - 8\lambda - a + 2a^2)(116a - 32\lambda + 88a\lambda - 179a^2 + 38a^3 - 36)}{64(1-a)(3a-2)(4a-3)^2} \end{aligned}$$

respectively.

Based on the monotonicity of price and profit differentials (see the Appendix), and restricting the analysis to  $a \in [0, 0.5)$  satisfying the SOCs (see also the Appendix), the following results hold:

**Result 3:** *There exists a unique threshold of cost savings  $\hat{\lambda}$  such that the distant merger does not affect prices. This threshold is negative.*

It is straightforward to check that  $\hat{\lambda} = \frac{1}{4} \frac{(2a-1)a}{2-3a}$  solves for  $g(\lambda, a) = l(\lambda, a) = 0$ . Note however that  $\hat{\lambda} \leq 0$  for any  $a \in [0, 0.5)$ . This leads therefore to:

**Result 4:** *Both the insider and the outsider platforms lower their prices regardless of the amount of cost savings generated by the distant merger, but the latter is internally profitable only for high enough cost savings, i.e.  $\lambda \in [\lambda^*, \bar{\lambda}]$ .*

*See proof in the Appendix.*

As far as the post-merger pricing behavior is concerned, note first that for  $a = 0$  and  $\lambda = 0$ , the standard outcome from one-sided spatial oligopoly literature still holds: the merger

between distant firms does not affect prices, since the two merging firms have no captive demand between them to exploit. In turn, as soon as the indirect externality is present, the merger's efficiency gains no longer matter, since in that case the post-merger prices fall anyway. In other words, the distant merger yields an extreme case of complementarity between the market's two-sidedness and the merger's efficiency gains, to the extent that only the first matters for the pricing behavior of platforms. Note as well that the type of merger appears to be crucial for the relative importance of the cross-group externality: for the pricing behavior the latter is clearly dominant in the case of a merger between distant platforms.

On the other hand, Result 4 equally establishes that for the 'distant' merger to take place, the insider platforms need enough cost savings. This is easily explained by looking into the merger's effect on the outsiders' profit. Recall first that both the indirect externality and the cost savings provide incentives for the insiders to lower their prices. The outsider platforms will do the same, but by less, thus losing demand in the process. The negative impact on their profit is unambiguous, all the more so that our four-firm setting implies that each outsider faces a price drop both to the left and to the right of its location. This negative profit effect for the outsiders (which do not however exit the market after the merger -see the Appendix) basically extends Brito's (2005) result to two-sided markets, and is actually magnified here by the fact that it occurs on both sides of the market as well.

Finally, the policy implication of Results 3 and 4 are quite straightforward: the efficiency defense is redundant in case of distant merger, since this type of merger is pro-competitive whenever submitted. This is again a direct extension of the one-sided conclusion to the two-sided case, pointing at the fact that the insights from one-sided markets can, in some cases, safely apply to two-sided situations.

One final observation is nevertheless worth making: what really matters for the competitive analysis of mergers on two-sided markets with differentiated platforms is the combination of 'merger type' and degree of 'two-sidedness'. To see this, consider the case of a relatively weak indirect externality: if the merger involves neighbor firms, meaning closely substitutable platforms, then the efficiency defense is necessary to make sure that customers are not harmed. However, if the merger involves distant, or less substitutable platforms, our analysis indicates that competition agencies can spare the money and time usually spent on efficiency defense arguments, since this merger is pro-competitive.

## 4 Simulating a more general case

In this section we are going to extend the results obtained under perfect symmetry between sides by allowing for asymmetry in the reciprocal valuation, i.e.  $\beta \in [0, 1]$ . Relaxing the perfect symmetry assumption turns the price differentials into three-parameter functions, making necessary Mathematica simulations to derive results (see the Technical Appendix). We begin by looking into how the valuation asymmetry between sides affects the efficiency gains thresholds necessary to keep prices constant, then go on to examine its impact on the scope of the efficiency defense.

### 4.1 Main results

Recall that the main purpose of our analysis is to determine the impact of the amount of cost savings on the change in prices due to the merger. In other words, we examine the possible scope for an efficiency defense. However, consider the situation where customers would benefit from the merger, but the latter is not profitable for the merging firms. In such a case the merger would likely not be submitted in the first place, and as a result there would be no occasion for the efficiency defense to apply.

Lemma 1 below discusses the merger's impact on the platforms' profits, so as to draw proper conclusions for the applicability of the efficiency defense.

**Lemma 1** *For each side of the market, and for each type of merger (between adjacent or distant platforms), there exists a non degenerate interval of cost savings such that the merger is internally profitable without causing the outsiders' exit from the market.*

**Proof:** see Tables xxx in the Technical Appendix B.

To put it short, we restrict for the rest of the analysis to the range of merger cost savings on each side of the market such that it is profitable for the insiders to merge without forcing the market exit of the outsiders.<sup>17</sup>

Note that Lemma 1 provides a generalization of previous results obtained in the particular case of perfect symmetry between sides. For adjacent platforms, Mathematica simulations yield

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<sup>17</sup>Whatever the type of merger, the insiders' profit on each side of the market is convex in efficiency gains, with two real roots, one of which is always strictly negative. This other root consequently represents the lower bound of permissible values for the efficiency gains parameter on either side of the market. The upper bound of this interval, one for each side, will be given by the minimum value between two thresholds: the efficiency gains level from merger such that the outsiders serve a zero residual demand, and the efficiency gains level such that the outsider's margin is zero.

that the merger is always internally profitable for any  $\beta \in [0, 1]$  and  $a \in [0, 0.5)$ . In other words, regardless of the difference in valuation between sides, and whatever the cost savings obtained through merger, the latter is profitable for the merging platforms as long as the indirect externality is "weak".<sup>18</sup> This turns out to be quite intuitive to the extent that a "low" degree of two-sidedness allows a direct extension of the benchmark conclusion obtained on one-sided markets: adjacent firms sharing market power over the captive demand located between them will find it profitable to merge thanks to this market power effect. The simulations behind Lemma 1 show that this not only holds for a two-sided context as long as it can be safely "approximated" by a one-sided framework (weak cross-group externality), but also that the asymmetry in valuation between sides does not overturn this result. Moreover, the same conclusion obtains as well for some higher values of the cross-group externality ( $a > 0.5$ ), provided that the valuation asymmetry between sides is high enough ( $\beta$  close enough to 0). In other words, for more "two sided" markets, the adjacent merger remains always profitable, meaning it does not require cost savings in order to take place, provided that one of the sides hardly values the other. This is actually quite intuitive again, given the multiplicative form capturing the total intensity of the link between the two market sides. More precisely, the latter is given by the product between the indirect externality and the valuation of one side by the other, which results in a certain substitutability between the two. Hence, a "strong" cross-group externality coupled with a high valuation asymmetry, i.e.  $a > 0.5$  with  $\beta$  close enough to 0, boils down to the same "overall" two-sidedness as a "weak" cross-group externality coupled with any asymmetry between the reciprocal but positive valuations of the two sides, i.e.  $a \in [0, 0.5)$  with  $\beta \in [0, 1]$ . Note that this intuition is robust, since it is further confirmed by the additional outcome of the Mathematica simulations. These show that the adjacent merger requires enough cost savings to occur in case of a strong enough indirect externality coupled with a low enough valuation asymmetry between sides, i.e. for  $a > 0.5$  and  $\beta$  close enough to 1. This basically generalizes Result 2 obtained in the particular case of perfect symmetry between sides, i.e.  $\beta = 1$ , according to which the strong two-sidedness prevents the profitable exploitation of market power after merger by favoring customers instead of rival platforms<sup>19</sup>, making the efficiency gains thus necessary for the merger to take place.

As far as the distant merger is concerned, it requires a minimum amount of cost savings in order to be internally profitable whatever the asymmetry in valuation between sides ( $\beta \in$

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<sup>18</sup>More precisely, in this case both roots of the insiders' post-/pre-merger profit differential as a convex function of efficiency gains are negative.

<sup>19</sup>This was previously called the "reversed" business stealing effect.

$[0, 1]$ ) and whatever the intensity of the cross-group externality, "weak" ( $a < 0.5$ ) or "strong" ( $a > 0.5$ )<sup>20</sup> Again, this is an extension of the one-sided framework result, according to which the lack of captive demand between the two insiders prevent them from exploiting market power through their merger, pointing at the efficiency gains as a valid explanation for the merger. But the competitive pressure exerted by the outsiders located between the insiders, on each of their sides, indicates that very small price decreases would not shift enough demand away from the outsiders. Important price decreases are then necessary to steal away the outsiders' captive customers, located close to them, therefore it takes enough cost savings to lower the price sufficiently and thereby make the merger profitable. The fact that neither the extent of the valuation asymmetry between sides nor the intensity of the indirect externality prevent the robustness of the one-sided conclusion shows that the type of merger considered, here the distant one, is determinant.

This sort of insight will be further confirmed below by Lemma 2, which deals with the merger cost savings that are necessary in order for the merger to keep prices constant.

**Lemma 2** *For each type of merger (between adjacent or distant platforms), there exists a unique threshold of cost savings on each side of the market such that the insiders' (outsiders') price is the same as before merger.*

**Proof:** *The post-/pre-merger price differentials for the insiders/outsider platforms are linear in the efficiency gains parameter - see their respective expressions in the Technical Appendix A.*

Lemma 2 only states the existence of the efficiency gains threshold keeping the Insiders'/Outsiders' prices constant on each side of the market, and as such is not informative for the scope of an efficiency defense in this more general setting. Proposition 1 below provides this result:

**Proposition 1** *Consider different but positive reciprocal valuations between the two market sides, i.e.  $\beta \in [0, 1]$ ; then*

*(i) for the adjacent merger, the insiders' and outsiders' constant-price efficiency gains thresholds for both sides belong to the intervals identified in Lemma 1 if the indirect externality is "weak" ( $a < 0.5$ ) whatever the valuation asymmetry between sides (i.e.  $\beta \in [0, 1]$ ), or if the indirect externality is "strong" and the valuation asymmetry between sides is high ( $a > 0.5$  and  $\beta$  close to 0); however, both thresholds are generally negative on both sides of the market if the*

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<sup>20</sup>More precisely, the corresponding Mathematica simulations yield that one of the roots of the insiders' profitability differential is always positive.

indirect externality is "strong" and the asymmetry between sides is low ( $a > 0.5$  and  $\beta$  close to 1).

(ii) for the distant merger, the constant-price efficiency gains thresholds are generally lower than the lower bound of the intervals in Lemma 1 for all prices on both sides.

**Proof:** see Tables xxx in the Technical Appendix.

Proposition 1 makes clear the scope for an efficiency defense for each side of the market and for each type of merger, by restricting the permissible ranges of parameters to those where the merger's efficiency gains will prevent price increases. Similarly to Lemma 1, Proposition 1 offers a generalization of results obtained in the particular case of perfect symmetry when allowing for different reciprocal valuations between the market's sides.

More precisely, for the adjacent merger the efficiency defense is still necessary in case of global "low" two-sidedness, but may become redundant in case of overall high "two-sidedness", whereas the efficiency defense remains throughout redundant for the distant merger. In other words, the introduction of valuation asymmetry between sides does not qualitatively overturn the results obtained under the simplifying assumption of perfect symmetry. The change merely concerns the definition of global or overall two-sidedness.<sup>21</sup> Accordingly, a strong cross-group externality coupled with a high valuation asymmetry, i.e.  $a > 0.5$  and  $\beta$  close enough to 0, boils down to the same "overall" two-sidedness as a weak cross-group externality coupled with any asymmetry between the reciprocal but positive valuations of the two sides, i.e.  $a \in [0, 0.5)$  with  $\beta \in [0, 1]$ . In both circumstances the efficiency defense is necessary for the adjacent merger according to Proposition 1. This is straightforward given that both combinations correspond to a "weak" global two-sidedness, approximating therefore the one-sided case of a merger between neighbor firms. Recall that in this case it takes high enough cost savings to overwhelm the market power effect of the merger and thus for the customers to benefit from the latter.<sup>22</sup> In turn, the combination of strong indirect externality and low valuation asymmetry between sides, i.e.  $a > 0.5$  and  $\beta$  close to 1 replicates the opposite case of a "strong" overall two-sidedness, for which the efficiency defense becomes redundant for the adjacent merger, because regardless of the cost savings obtained through merger, the insiders prefer to lower their price instead of exploiting market power.

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<sup>21</sup>This, as before mentioned, is quite intuitive given the multiplicative form capturing the intensity of the link between the two market sides: the level of global two-sidedness is given by the product between the level of the cross-group externality and that of the valuation of the presence of the other side.

<sup>22</sup>Note that eventually all customers throughout each side of the market benefit from the corresponding insiders' price drop, since the latter is replicated by the outsiders due to the strategic complementarity.

The same conclusion holds for the distant merger, whatever the intensity of the cross-group externality: the efficiency defense remains redundant in case of different valuations between sides. To put it differently, the insiders of a distant merger will always lower prices regardless of merger's cost savings and of the degree of overall two-sidedness. The intuition is simple. Recall that such a merger cannot involve a unilateral market power effect, since the insiders do not share captive demand among them. Therefore the only profitable strategy for the insiders is to lower prices, although there's no need for cost savings to provide incentives in that direction, given the demand gain obtained thanks to the market's two-sidedness.

The bottom line is therefore that the same qualitative conclusions obtain for the scope of the efficiency defense when we allow for different positive valuations between the market's sides, including the fact that the type of merger remains crucial. More precisely, the efficiency defense is redundant in the case of distant merger, whatever the degree of market two-sidedness. It is in turn necessary for the adjacent merger in case of low two-sidedness, but may become redundant as well for high two-sidedness.

We can therefore sum up the policy implications of Proposition 1 in the following:

**Corollary 1** *The scope of the efficiency defense on two-sided markets is robust to the assumption of asymmetric valuation between sides. The scope of the efficiency defense only depends on the market's two-sidedness only if the merger concerns adjacent platforms. The efficiency defense is only necessary for the adjacent merger and for weak two-sidedness.*

In other words, the same merger policy as on one-sided markets can be safely applied to two-sided markets as long as the two-sidedness is weak. In contrast, with strong two-sidedness, the type of merger becomes crucial to justify the use of the efficiency defense.

## 4.2 Additional results

In this final section we shall discuss the impact on the efficiency defense of the difference in reciprocal valuation between sides ( $\beta$ ). Therefore we are going to restrict these additional results to the case of the adjacent merger, the only one for which there is scope for an efficiency defense.

Mathematica simulations (see Tables \*\*\* in the Technical Appendix B) show that the difference between the insiders' and outsiders' constant-price efficiency gains thresholds on each side of the market, as well as the differences between the insiders'/outsiders' respective thresholds between the two sides are always non null. In other words, the following holds:

**Result 5:** *For the adjacent merger, if the cross-group externality is "weak", i.e.  $a \in (0, 0.5)$ , then the insiders lower their price first on side 2, i.e. for a lower level of merger cost savings than on side 1, whereas the outsiders lower their price first on side 1, i.e. for a lower level of merger cost savings than on side 2. Also, the stronger the valuation asymmetry between sides ( $\beta \rightarrow 0$ ), the larger the gap in price decreases between sides for both the insiders and the outsiders.*

To grasp the intuition for this, recall that side 1 values the presence of side 2 more than what side 2 values the presence of side 1. The insiders, obtaining efficiency gains through the merger, start lowering their price "first", i.e. for a lower amount of cost savings, on the side that will give them the highest return in terms of additional cross-sides demand, here side 2. The strategic reason is that the latter is more valued by side 1, dropping the price "first" on side 2 gives them a higher gain on side 1 than they would obtain on side 2 by dropping "first" on side 1. The outsiders' reply takes into account that they could not match this price drop due to their lack of cost savings. So, in order to minimize the demand loss inflicted by the insiders, the outsiders will respond by dropping their price "first" on the side that brings the insiders a higher relative demand gain (side 1); only "afterwards" (for a higher amount of merger cost savings), will the outsiders start to lower their price on the other side (side 2 here). Furthermore, these side-differentiated price decreases are enhanced for both the insiders' and the outsiders' in case of stronger valuation asymmetry between the market sides.

However, the following result also holds:

**Result 6:** *For the adjacent merger, if the cross-group externality is "strong",  $a > 0.5$ , and there is a large enough difference in valuation between sides, i.e.  $\beta$  close enough to 0, the outsiders may mimic the insiders' strategy and also lower their price first on side 2, i.e. for a lower level of merger cost savings than on side 1.*

This last result shows that, with a difference in the reciprocal valuation between sides and a high enough indirect externality, it is possible that on the side that values more the presence of the other side (here, side 1 values more side 2 than what side 2 values side 1), customers pay higher prices to both insiders and outsiders for "longer", i.e. the efficiency gains necessary for all prices to drop are higher than on the other side. This change in the outsiders' behavior is due to the combined effect of a strong cross-group externality and high difference in valuation between sides, which increases the opportunity cost of preserving demand on the side on which the insiders are the most price aggressive following the merger. As a result, the outsiders' loss-

minimizing strategy is to mimic the insiders. Importantly, this implies that all prices on the side that values less the presence of the other side may fall for lower values of the merger’s efficiency gains than on the higher-valuating side, meaning that a price drop on one side of the market from all firms is compatible with a simultaneous price increase on the other side. Therefore the merger’s net effect on customers would likely involve balancing one side’s gain against the other side’s loss, which is still an open question for competition authorities.

## 5 Concluding remarks

This paper studied bilateral horizontal mergers on a two-sided market with four symmetrically differentiated platforms. Assuming positive and reciprocal valuation between the two sides, we provided a theoretical analysis of the merger’s price effect based on the amount of cost savings it generates and the degree of market two-sidedness in order to discuss the merger control policy implications. In short, the two-sidedness makes the efficiency defense redundant if the merger takes place between distant platforms. In the case of adjacent merging platforms, however, the efficiency defense is still necessary if the two-sidedness is relatively weak. The table below summarizes our results:

scope of the efficiency defense	weak two-sidedness	strong two-sidedness
adjacent merger	necessary	possibly redundant
distant merger	redundant	redundant

We further show that these conclusions are robust to asymmetric reciprocal valuations between the two sides, and that the type of merger, in terms of closeness or overlap between the insiders, is determinant for the scope of the efficiency defense, much as on one-sided markets. Therefore the main takeaway of the paper is that as long as the two-sidedness is low enough, the same merger control can be safely applied as on one-sided markets. We leave for future research the discussion of alternative assumptions, such as usage fees instead of access fees, or that of non-symmetric differentiation between platforms, or non-localized competition between them.

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## 6 Appendix

### 6.1 Proofs for the illustrative example

- Merger between adjacent platforms

#### Hessian matrices

Whenever the SOCs are satisfied the Hessian matrices for all profit functions are negative definite (ND). We define below the Hessian matrix  $H^{A+B}$  for the merged firm and  $H^{C,D}$  for the outsider profits:

$$H^{A+B} = \frac{N}{1-4a^2} \begin{bmatrix} \frac{1}{-a^2+1} (5a^2 - 4) & 3\frac{a}{-a^2+1} (2a^2 - 1) & 1 & 2a \\ 3\frac{a}{-a^2+1} (2a^2 - 1) & \frac{1}{-a^2+1} (5a^2 - 2) & 2a & 1 \\ 1 & 2a & \frac{1}{-a^2+1} (5a^2 - 2) & 3\frac{a}{-a^2+1} (2a^2 - 1) \\ 2a & 1 & 3\frac{a}{-a^2+1} (2a^2 - 1) & \frac{1}{-a^2+1} (5a^2 - 2) \end{bmatrix}$$

and  $H^{C,D} = \frac{1}{(1-a^2)(1-4a^2)} \begin{bmatrix} 5a^2 - 2 & 3a(2a^2 - 1) \\ 3a(2a^2 - 1) & 5a^2 - 2 \end{bmatrix}$ .

In order for  $H^{A+B}$  and  $H^{C,D}$  to be ND the following principal minor conditions need to be fulfilled:

- for the merged firm:  $|H_1^{A+B}| = \frac{5a^2-2}{4a^4-5a^2+1} < 0$ ,  $|H_2^{A+B}| = -\frac{9a^2-4}{4a^4-5a^2+1} > 0$ ,  $|H_3^{A+B}| = -\frac{11a^2-6}{4a^4-5a^2+1} < 0$ ,  $|H_4^{A+B}| = \frac{16a^2-9}{4a^4-5a^2+1} > 0$ ;
- for each outsider:  $|H_1^{C,D}| = \frac{5a^2-2}{4a^4-5a^2+1} < 0$ ,  $|H_2^{C,D}| = -\frac{9a^2-4}{4a^4-5a^2+1} > 0$ .

One can relatively easily check that the SOCs are satisfied for  $a < 0.5$  or  $0.75 < a < 1$ .

#### Proof of Result 2

Recall that the price differences for the insiders ( $A$  and  $B$ ) and the outsiders ( $C$  and  $D$ ) respectively write  $f(\delta, a) = \frac{(3-5a)(6a\delta-4\delta-2a+1+a^2)}{(4-6a)(5-8a)}$  and  $h(\delta, a) = \frac{1}{2} \frac{(1-2a)(6a\delta-4\delta-2a+1+a^2)}{(2-3a)(5-8a)}$ , with  $f(\delta, a) = h(\delta, a) = 0$  for  $\tilde{\delta} = \frac{(a-1)^2}{2(2-3a)} > 0$ .

One has that  $\frac{\partial}{\partial \delta} f(\delta, a) = \frac{3-5a}{8a-5} < 0$  for  $a \in [0, 0.5) \cup (0.75, 1)$  and  $\frac{\partial}{\partial \delta} h(\delta, a) = \frac{1-2a}{8a-5} < 0$  for  $a \in [0, 0.5) \cup (0.75, 1)$  as well. Note also that for  $\delta = 0$  the price differences write  $f(0, a) = \frac{1}{2} \frac{(1-a)^2}{2-3a} \frac{3-5a}{5-8a}$  and  $h(0, a) = \frac{1}{2} (1-a)^2 \frac{1-2a}{(2-3a)(5-8a)}$ . Therefore  $f(0, a) < 0$  and  $h(0, a) < 0$  if  $a \in (0.75, 1)$ , but  $f(0, a) \leq 0$  and  $h(0, a) \leq 0$  when  $\delta \geq \tilde{\delta}$  if  $a \in [0, 0.5)$ .

Recall that the merger profitability differential for the insider platforms writes  $\tilde{\Pi}^{A+B} - 2\Pi^* = \frac{4(2-3a)^3 \delta^2 + 4(4-7a)(1-a)(2-3a)^2 \delta + (19a^2 - 23a + 7)(1-a)^3}{2(1-a)(8a-5)^2(2-3a)}$ . We have that  $\tilde{\Pi}^{A+B} - 2\Pi^* = 0$  if  $\delta = \tilde{\delta}_1$  or  $\delta = \tilde{\delta}_2$ , where  $\tilde{\delta}_1 = \frac{1}{2} \frac{a-1}{(3a-2)^2} \left( (2-3a)(4-7a) + \sqrt{(2a-1)(3a-2)(8a-5)^2} \right)$  and  $\tilde{\delta}_2 = \frac{1}{2} \frac{a-1}{(3a-2)^2} \left( (2-3a)(4-7a) - \sqrt{(2a-1)(3a-2)(8a-5)^2} \right)$ .

We have that:

- if  $a \in [0, 0.5)$ , then  $\tilde{\delta}_1 < 0$  and  $\tilde{\delta}_2 < 0$ , that is  $\tilde{\Pi}^{A+B} - 2\Pi^* > 0$ .
- if  $a \in (0.75, 1)$ , then  $0 < \tilde{\delta}_2 < \bar{\delta}$  and  $\tilde{\delta}_1$  remains negative, that is  $\tilde{\Pi}^{A+B} - 2\Pi^* > 0$  if  $\Delta > \tilde{\delta}_2$ .

The merger's impact on the outsiders' profits is measured by

$$\tilde{\Pi}^O - \Pi = (1-2a) \frac{(4-6a)^2 \delta^2 - 12(1-a)(3a-2)^2 \delta + (1-a)^3(11-17a)}{4(1-a)(2-3a)(8a-5)^2}.$$

We have that  $\tilde{\Pi}^{C,D} - \Pi^* = 0$  if  $\delta = \tilde{\delta}_1^O$  or  $\delta = \tilde{\delta}_2^O$ , where  $\tilde{\delta}_1^O = \frac{(1-a)^2}{4-6a} \equiv \tilde{\delta}$  and  $\tilde{\delta}_2^O = \frac{1-a}{4-6a} (11-17a)$ .

We conclude that:

- if  $a \in [0, 0.5)$ ,  $0 < \tilde{\delta}_1^O < \bar{\delta} < \tilde{\delta}_2^O$  and then  $\tilde{\Pi}^O - \Pi^* \geq 0$  for  $\delta \leq \tilde{\delta}$  and  $\tilde{\Pi}^O - \Pi^* \leq 0$  for  $\tilde{\delta} \leq \delta \leq \bar{\delta}$ ;
- if  $a \in (0.75, 1)$ ,  $\tilde{\delta}_1^O > 0 > \bar{\delta} > \tilde{\delta}_2^O$  and then  $\tilde{\Pi}^O - \Pi^* \leq 0$  for  $\delta \leq \bar{\delta}$ .

### • Merger between distant platforms

#### Hessian Matrices:

- for the merged firm ( $A + C$ ):

$$H^{A+C} = \frac{1}{(1-a^2)(1-4a^2)} \begin{bmatrix} 5a^2 - 2 & 3a(2a^2 - 1) & -3a^2 & -a(2a^2 + 1) \\ 3a(2a^2 - 1) & 5a^2 - 2 & -a(2a^2 + 1) & -3a^2 \\ -3a^2 & -a(2a^2 + 1) & 5a^2 - 2 & 3a(2a^2 - 1) \\ -a(2a^2 + 1) & -3a^2 & 3a(2a^2 - 1) & 5a^2 - 2 \end{bmatrix}$$

- and for the outsiders ( $B$  or  $D$ ):

$$H^O = \frac{1}{(1-a^2)(1-4a^2)} \begin{bmatrix} 5a^2 - 2 & 3a(2a^2 - 1) \\ 3a(2a^2 - 1) & 5a^2 - 2 \end{bmatrix}.$$

Leading to principal minors and conditions:

-for the insider platforms:  $\left|H_1^{A+C}\right| = -\frac{2-5a^2}{4a^4-5a^2+1} < 0$ ,  $\left|H_2^{A+C}\right| = \frac{4-9a^2}{4a^4-5a^2+1} > 0$ ,  $\left|H_3^{A+C}\right| = -\frac{8}{4a^4-5a^2+1} < 0$ ,  $\left|H_4^{A+C}\right| = \frac{16}{4a^4-5a^2+1} > 0$ ;  
- for the outsiders:  $\left|H_1^{B,D}\right| = -\frac{2-5a^2}{4a^4-5a^2+1} < 0$ ,  $\left|H_2^{B,D}\right| = \frac{4-9a^2}{4a^4-5a^2+1} > 0$ .  
The SOCs are satisfied for  $a < 0.5$ .

#### Proof of Result 4

Recall that the price differences for the insiders ( $A$  and  $C$ ) and the outsiders ( $B$  and  $D$ ) respectively are given by  $g(\lambda, a) = \frac{(4-5a)(12a\lambda-8\lambda-a+2a^2)}{8(3a-2)(4a-3)}$  and  $l(\lambda, a) = \frac{(1-a)(12a\lambda-8\lambda-a+2a^2)}{4(3a-2)(4a-3)}$ .

Recall also that  $\hat{\lambda} = \frac{1}{4} \frac{(2a-1)a}{2-3a}$  solves for  $g(\lambda, a) = l(\lambda, a) = 0$ , where  $\hat{\lambda} \leq 0$  for any  $a \in [0, 0.5)$ .

Moreover,  $\frac{\partial}{\partial \lambda} g(\lambda, a) = \frac{4-5a}{8a-6}$  and  $\frac{\partial}{\partial \lambda} l(\lambda, a) = \frac{1-a}{4a-3}$ , therefore both price differences are decreasing in  $\lambda$  for  $a \in [0, 0.5)$ . Therefore one has that for all  $z \in [0, 0.5t)$ ,  $g < 0$  and  $l < 0$  as well.

The profitability for the merging platforms is given by

$$\hat{\Pi}^{A+C} - 2\Pi^* = \frac{\begin{pmatrix} 72a - 576\lambda + 3104a\lambda - 394a^2 + 785a^3 - 676a^4 + 212a^5 + 128\lambda^2 \\ -576a\lambda^2 - 6256a^2\lambda + 5592a^3\lambda - 1872a^4\lambda + 864a^2\lambda^2 - 432a^3\lambda^2 \end{pmatrix}}{64(1-a)(3a-2)(4a-3)^2}.$$

We have that  $\hat{\Pi}^{A+C} - 2\Pi^* = 0$  if  $\lambda = \lambda_1$  or  $\lambda = \lambda_2$ , where

$$\lambda_1 = \frac{1}{4(2-3a)^2} \left( (2-3a)(26a^2 - 43a + 18) + 2\sqrt{(1-a)(2-3a)(3-4a)^2(35a^2 - 49a + 18)} \right)$$

$$\text{and } \lambda_2 = \frac{1}{4(2-3a)^2} \left( (2-3a)(26a^2 - 43a + 18) - 2\sqrt{(1-a)(2-3a)(3-4a)^2(35a^2 - 49a + 18)} \right).$$

It can be easily checked that for  $a < 0.5$ ,  $\lambda_1 > \lambda_2 > 0$  and  $\lambda_1 > \bar{\lambda} > \lambda_2$ , with  $\hat{\Pi}^{A+C} - 2\Pi^* \geq 0$  for  $\lambda_1 \geq \lambda \geq \lambda_2$ .

The merger's impact on the outsiders' profits is measured by

$$\hat{\Pi}^O - \Pi^* = \frac{(12a\lambda-8\lambda-a+2a^2)(116a-32\lambda+88a\lambda-179a^2+38a^3-36)}{64(1-a)(3a-2)(4a-3)^2}.$$

One has that  $\hat{\Pi}^{B,D} - \Pi^* = 0$  if  $\lambda = \lambda_1^O$  or  $\lambda = \lambda_2^O$ , where  $\lambda_1^O = -a \frac{2a-1}{12a-8} \equiv \hat{\lambda} < 0$ , and  $\lambda_2^O = \frac{1}{(2-3a)(4-5a)} (116a - 119a^2 + 38a^3 - 36) < 0$  also for  $a \in [0, 0.5)$ .

One can easily check that  $\hat{\Pi}^O - \Pi^* < 0$  for all  $a \in [0, 0.5)$ .

## 6.2 Examples of Mathematica simulations for the asymmetric valuations case

Below we provide the simulations tables for the efficiency gain thresholds keeping prices constant on each side of the market for the insider and outsider platforms respectively. The values of the simulated expressions have been obtained with Maple - see the Technical Appendix for the detailed derivation.

"A+B merger - efficiency gains thresholds keeping prices constant on each side, for insiders and outsiders respectively"

"efficiency gains threshold for insiders - side 1; b denotes beta; rows: a values; columns: beta values"

Table1

$$\left( \left( -\frac{1}{2} \right) * (30 + 18 * b^5 * a^7 + 45 * a^7 * b^3 + 57 * b^4 * a^7 - 33 * b^2 * a^6 - 24 * b^4 * a^6 - 100 * b^3 * a^6 - 8 * a - 9 * b * a - 10 * a^2 - 153 * b * a^2 + 62 * b^2 * a^3 + 41 * a^3 * b + 3 * b^3 * a^3 - 15 * b^4 * a^5 - 6 * b^2 * a^2 - 72 * a^5 * b^2 - 122 * b^3 * a^5 + 37 * b * a^4 + 24 * b^3 * a^4 + 235 * b^2 * a^4) \right) / \left( (-4 + 9 * b * a^2) * (15 - 7 * a^2 - 4 * a + 3 * b * a - 3 * b^2 * a^2 - 45 * b * a^2 + 9 * a^3 * b + 19 * b^2 * a^4 + 15 * b * a^4 - 8 * b^2 * a^3 + 6 * b^3 * a^4) \right), \{a, \{0.001, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.786, 0.8, 0.9, 0.999, 1.1, 1.2\}\}, \{b, \{1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0\}\} ]$$

0.249875	0.249876	0.249894	0.249906	0.249919	0.249938	0.249944	0.249969	0.249981	0.25
0.228571	0.228778	0.231716	0.233869	0.236067	0.239448	0.240598	0.245305	0.247725	0.251437
0.222727	0.222953	0.226299	0.228903	0.231675	0.236136	0.237701	0.244331	0.247861	0.253417
0.225	0.225039	0.226375	0.228133	0.230473	0.234943	0.236665	0.244579	0.249099	0.256515
0.249504	0.248519	0.239005	0.235961	0.235163	0.237133	0.238467	0.246446	0.251753	0.261056
-5.26316	0.387239	0.29562	0.267962	0.253677	0.245673	0.245277	0.250766	0.256544	0.267857
-0.125	-0.133858	-0.611011	1.16686	0.431755	0.290735	0.277815	0.265382	0.270495	0.284884
-0.0639609	-0.0676751	-0.199079	-1.13616	0.671218	0.318472	0.295768	0.271603	0.275989	0.291015
-0.05	-0.052727	-0.14429	-0.551794	0.943726	0.335294	0.30539	0.274507	0.278485	0.293716
-0.00714286	-0.00705782	-0.0133747	-0.0413894	-0.159797	0.713068	0.425742	0.311393	0.306226	0.320681
-5.01504 × 10 <sup>-7</sup>	0.00133944	0.0182912	0.0224251	0.00860289	-0.257433	-4.55774	1.01093	0.386867	0.374191
-0.00384615	-0.00251738	0.00468422	-0.0864685	0.123345	0.0351978	-0.0424837	0.267945	-1.55033	0.534038
-0.0125	-0.0108562	0.00740197	0.0102045	-0.012939	0.691143	0.198001	0.405029	0.134066	6.25



"efficiency gains threshold for outsiders - side 1; b denotes beta; rows: a values; columns: beta values"

Table[ $\left(\frac{-1}{2}\right) * (10 + 18 * b^5 * a^7 + 30 * b^4 * a^7 - 15 * b^2 * a^6 - 24 * b^4 * a^6 - 19 * b^3 * a^6 + 4 * a - 13 * b * a - 2 * a^2 - 52 * b * a^2 + 63 * b^2 * a^3 - 18 * a^3 * b + 3 * b^3 * a^3 - 15 * b^4 * a^5 - 6 * b^2 * a^2 + 20 * a^5 * b^2 - 92 * b^3 * a^5 + 11 * b * a^4 + 24 * b^3 * a^4 + 73 * b^2 * a^4) /$

$\left((-4 + 9 * b * a^2\right) * (5 - 3 * b^2 * a^2 - 18 * b * a^2 + 10 * b^2 * a^4 + 2 * a - 5 * a^3 * b + 7 * b^2 * a^3 + 6 * b^3 * a^4 - 4 * b * a)$ ,

{a, {0.001, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.786, 0.8, 0.9, 0.999, 1.1, 1.2}},

{b, {1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0}}]

0.249875	0.249876	0.249894	0.249906	0.249919	0.249938	0.249944	0.249969	0.249981	0.25
0.228571	0.228798	0.231884	0.234001	0.236053	0.239021	0.239984	0.243718	0.24552	0.248148
0.222727	0.223011	0.226778	0.229292	0.231701	0.235172	0.236299	0.240693	0.242833	0.245982
0.225	0.22521	0.22764	0.229176	0.230739	0.233247	0.234134	0.237926	0.239948	0.243103
0.249504	0.253085	0.243657	0.239157	0.236401	0.234611	0.234486	0.235658	0.236991	0.239621
0.4	0.388545	0.290964	0.434758	0.260646	0.242601	0.239568	0.234184	0.23399	0.235484
-0.125	-0.134321	-0.628546	1.18738	0.422324	0.305663	0.271286	0.234958	0.229943	0.228365
-0.0639609	-0.0680548	-0.209654	-1.18635	0.678025	0.272295	0.293468	0.236027	0.229122	0.226499
-0.05	-0.0530866	-0.153978	-0.583967	0.964188	0.304918	0.310144	0.236576	0.228825	0.225758
-0.00714286	-0.00735176	-0.0221675	-0.060388	-0.196484	0.728269	0.420288	0.243528	0.227211	0.220221
-5.01504 × 10 <sup>-7</sup>	0.000478553	0.0024397	-0.005109	-0.03219	-0.341214	-5.10812	0.25938	0.226898	0.214347
-0.00384615	-0.00268809	-0.00864492	0.00392283	-0.000139505	-0.06075	-0.14636	0.297852	0.228813	0.207986
-0.0125	-0.0110333	0.016621	-0.0351853	-0.000931301	-0.0134643	-0.0374174	0.405054	0.234569	0.201351



"A+C merger - efficiency gains thresholds keeping prices constant on each side, for insiders and outsiders respectively"

"efficiency gains threshold for insiders - side 1; b denotes beta; rows: a values; columns: beta values"

Table1

$$\left( \frac{-1}{4} \right) * a * \left( 24 * b * a^2 - 40 * b^3 * a^5 + 228 * b^4 * a^6 - 6 * b^3 * a^3 - 24 * b + 180 * a^6 * b^3 - 56 * a^3 * b + 8 * a + 8 * b^2 * a^3 - 135 * a^4 * b^2 + 173 * b^2 * a^2 + 72 * a^6 * b^5 - 377 * b^3 * a^4 + 12 * b^3 * a^2 + 84 * b^2 * a^5 - 60 * a^4 * b^4 + 12 * a^5 * b^4 + 2 * b * a \right) / \left( (-4 + 9 * b * a^2) * (38 * a^4 * b^2 - 48 * b * a^2 - 13 * b^2 * a^3 + 9 * a^3 * b + 30 * a^4 * b + 12 * b^3 * a^4 - 6 * b^2 * a^2 - 14 * a^2 + 5 * b * a - 4 * a + 12) \right), \{a, \{0.001, 0.1, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.85, 0.9, 0.999, 1.15\}\}, \{b, \{1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0\}\}]$$

-0.000124937	-0.000123688	-0.000106197	-0.000093701	-0.000081201	-0.000062461	-0.000056211	-0.000031211	-0.000018711	4.16806 × 10 <sup>-8</sup>
			24	66	09	18	29	18	
-0.0117647	-0.0116492	-0.0100181	-0.00883763	-0.00764415	-0.00582942	-0.00521792	-0.00273868	-0.0014789	0.0004363
-0.0214286	-0.0212299	-0.0183757	-0.0162532	-0.0140615	-0.010644	-0.0094702	-0.00460134	-0.00206201	0.0018797
-0.0272727	-0.0270662	-0.0239109	-0.021366	-0.0185891	-0.0140049	-0.0123682	-0.00529274	-0.0014431	0.00471698
-0.025	-0.0249971	-0.0239877	-0.0222771	-0.0198475	-0.0150049	-0.0130981	-0.00414067	-0.00108573	0.00980392
-0.000496024	-0.00151237	-0.0110483	-0.0138639	-0.0142375	-0.0112752	-0.00951631	0.000636723	0.00732584	0.0191011
0.15	0.138409	0.0442197	0.0136106	-0.011584	0.0250674	0.0164757	0.0178177	0.0248631	0.0394737
-0.375	-0.384784	-0.8673	0.904589	0.164627	0.0233224	0.00448259	-0.0850971	-0.519518	0.25
-0.270455	-0.270041	-0.624488	-0.408752	-1.05918	0.147858	0.0703806	-0.0289431	-0.0680904	-0.238449
-0.257143	-0.256433	-0.250918	-0.125166	-0.439969	0.439193	0.157626	-0.0177549	-0.0505548	-0.137755
-0.250001	-0.248732	-0.234718	-0.231975	-0.242044	-0.542754	-4.80741	0.00508167	-0.0330864	-0.0836125
-0.257759	-0.255997	-0.233352	-0.220709	-0.213795	-0.234385	-0.270328	0.0790157	-0.0147314	-0.0594917

"efficiency gains threshold for insiders - side 2; b denotes beta; rows: a values; columns: beta values"

Table[ $\left(\frac{-1}{4}\right) * a * (72 * a^6 * b^2 + 173 * b * a^2 - 377 * a^4 * b^2 + 8 * b^2 * a - 40 * b^3 * a^5 - 56 * b^3 * a^3 + 24 * b^2 * a^2 + 12 * a^2 - 6 * a^3 * b + 228 * a^6 * b^3 + 8 * b^2 * a^3 + 84 * a^5 * b^4 - 60 * a^4 * b - 24 - 135 * b^3 * a^4 + 2 * b * a + 12 * b^2 * a^5 + 180 * b^4 * a^6) /$   
 $\left((-4 + 9 * b * a^2\right) * (38 * a^4 * b^2 - 48 * b * a^2 + 9 * b^2 * a^3 - 13 * a^3 * b + 12 * a^4 * b + 30 * b^3 * a^4 - 14 * b^2 * a^2 - 6 * a^2 - 4 * b * a + 5 * a + 12)$ ),  
{a, {0.001, 0.1, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.85, 0.9, 0.999, 1.15}},  
{b, {1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0}}]

-0.000124937	-0.000124938	-0.000124944	-0.000124948	-0.000124951	-0.000124953	-0.000124954	-0.000124954	-0.000124953	-0.000124952	-0.000124948
-0.0117647	-0.0117711	-0.0118522	-0.0118998	-0.0119391	-0.0119826	-0.011993	-0.011993	-0.0120148	-0.0120138	-0.011976
-0.0214286	-0.0214634	-0.0219062	-0.0221742	-0.0224039	-0.0226798	-0.022754	-0.022754	-0.022965	-0.0230201	-0.0230408
-0.0272727	-0.0273904	-0.0288826	-0.0297866	-0.0305693	-0.0315349	-0.0318045	-0.0318045	-0.0326382	-0.0329144	-0.0331597
-0.025	-0.0253696	-0.029262	-0.0325869	-0.0348417	-0.0375764	-0.0383349	-0.0406994	-0.041517	-0.0423313	-0.0423313
-0.000496024	-0.00183854	-0.0172276	-0.0253322	-0.0317304	-0.0389571	-0.0408654	-0.0465783	-0.0484894	-0.0504047	-0.0504047
0.15	0.138614	0.0417063	-0.00223857	-0.0096064	-0.0313662	-0.0362763	-0.0495599	-0.0535954	-0.0574766	-0.0574766
-0.375	-0.384963	-0.912329	0.989544	0.178791	-0.0228302	0.0186143	-0.0446607	-0.0558816	-0.0653409	-0.0653409
-0.270455	-0.274356	-0.29562	-0.425233	-1.21681	0.167511	0.0650508	-0.0289638	-0.0519716	-0.0683513	-0.0683513
-0.257143	-0.25952	-0.371806	-0.298521	-0.482301	0.539055	0.184353	-0.0122046	-0.0474448	-0.0690077	-0.0690077
-0.250001	-0.251327	-0.282171	-0.358857	-0.190443	-0.670094	-6.27534	0.199857	-0.0297996	-0.0682064	-0.0682064
-0.257759	-0.25854	-0.274651	-0.297797	-0.354316	-0.109725	-0.293971	0.075342	0.0763453	-0.0595358	-0.0595358

"efficiency gains threshold for outsiders - side 1; b denotes beta; rows: a values; columns: beta values"

Table1

$$\left( \frac{-1/4}{60 * b^4 * a^6 - 30 * a^4 * b^4 + 6 * b^3 * a^2 + 43 * b^2 * a^2 + 5 * b * a - 4 * a} \right) / \left( (-4 + 9 * b * a^2) * (-13 * b * a^2 + 2 * a - b * a - 5 * a^3 * b + 3 * b^2 * a^3 + 6 * b^3 * a^4 - 3 * b^2 * a^2 + 10 * a^4 * b^2) \right),$$

$$\{a, \{0.001, 0.1, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.85, 0.9, 0.999, 1.15\}\},$$

$$\{b, \{1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0\}\}]$$

-0.000124937	-0.000123688	-0.000106204	-0.000093711	-0.000081229	-0.0000625	-0.000056251	-0.000031289	-0.000018801	-8.32778 × 10 <sup>-8</sup>
-0.0117647	-0.0116523	-0.0110084	-0.00897038	-0.00786207	-0.00620935	-0.00566099	-0.00348002	-0.0023969	-0.00078125
-0.0214286	-0.021242	-0.0186357	-0.0167794	-0.0149265	-0.0121524	-0.0112288	-0.0075399	-0.00569863	-0.00294118
-0.0272727	-0.0270924	-0.0244992	-0.0225672	-0.0205702	-0.0174588	-0.0163926	-0.0119967	-0.00972811	-0.00625
-0.025	-0.0250406	-0.0250772	-0.0245396	-0.0235922	-0.0215141	-0.0206691	-0.0166581	-0.0143329	-0.0105263
-0.000496024	-0.00161254	-0.0133932	-0.0183912	-0.0213887	-0.0231569	-0.0231843	-0.0212394	-0.0193149	-0.0155703
0.15	0.138452	0.044103	-0.515855	-0.00691834	-0.0198804	-0.0222401	-0.0255322	-0.024673	-0.0214286
-0.375	-0.384853	-0.882447	0.926638	0.166243	0.0000927759	-0.00476431	-0.0298853	-0.0327117	-0.03125
-0.270455	-0.27286	-0.320262	-0.4176	-1.11882	0.148301	0.0725286	-0.0297471	-0.0377184	-0.0384309
-0.257143	-0.259507	-0.288043	-0.328444	-0.463857	0.459863	0.157865	-0.0279286	-0.0399639	-0.0421875
-0.250001	-0.284364	-0.283355	-0.288276	-0.312363	-0.614874	-5.35791	-0.0180188	-0.0435161	-0.04992
-0.257759	-0.253858	0.00653446	-0.377913	-0.315821	-0.324243	-0.362304	0.0881058	-0.0445762	-0.0623821

"efficiency gains threshold for outsiders - side 2; b denotes beta; rows: a values; columns: beta values"

Table[ $\left(\left(\frac{-1}{4}\right) * a * (36 * a^6 * b^2 + 43 * b * a^2 - 4 * b^2 * a + 22 * b^3 * a^3 - 3 * a^3 * b - 21 * b^2 * a^3 - 5 * b^3 * a^4 - 97 * a^4 * b^2 - 30 * a^4 * b + 6 * a^2 - 6 + 60 * a^6 * b^3 - 30 * a^5 * b^4 + 6 * b^2 * a^5 + 16 * b^3 * a^5 + 2 * b^2 * a^2 + 5 * b * a) \right) / \left( (-4 + 9 * b * a^2) * (-13 * b * a^2 + 10 * a^4 * b^2 + 2 * b * a - a + 3 - 5 * b^2 * a^3 + a^3 * b - 3 * a^2 + 6 * a^4 * b) \right)$ ,

{a, {0.001, 0.1, 0.2, 0.3, 0.4, 0.499, 0.6, 0.75, 0.85, 0.9, 0.999, 1.15}},

{b, {1, 0.99, 0.85, 0.75, 0.65, 0.5, 0.45, 0.25, 0.15, 0}}]

-0.000124937	-0.000124938	-0.000124942	-0.000124948	-0.000124955	-0.000124969	-0.000124974	-0.000125	-0.000125015	-0.000125042
-0.0117647	-0.0117683	-0.0118344	-0.0119007	-0.0119833	-0.0121381	-0.0121982	-0.0124815	-0.0126496	-0.0129355
-0.0214286	-0.0214522	-0.0218417	-0.0221894	-0.0225973	-0.0233266	-0.0236021	-0.0248737	-0.0256151	-0.0268657
-0.0272727	-0.0273663	-0.0287661	-0.0298776	-0.0310931	-0.0331309	-0.0338718	-0.0371717	-0.0390378	-0.0421296
-0.025	-0.025331	-0.0298395	-0.0329827	-0.0361271	-0.040951	-0.0426085	-0.0496023	-0.0533703	-0.059434
-0.000496024	-0.00184382	-0.0180714	-0.0273561	-0.0354689	-0.0463418	-0.0497647	-0.0630983	-0.0697868	-0.0801203
0.15	0.13857	0.0424743	0.00980772	-0.00535739	-0.0749827	-0.0694589	-0.084273	-0.0940144	-0.109091
-0.375	-0.384896	-0.900298	0.968136	0.174498	0.0148394	-0.00433664	0.0311356	-0.679349	-0.21875
-0.270455	-0.271425	-0.313674	-0.421633	-1.18176	0.160622	0.0676429	-0.030217	-0.00370333	5.05446
-0.257143	-0.256353	-0.269649	-0.317485	-0.474792	0.512128	0.174874	-0.0319611	-0.0302755	0.194318
-0.250001	-0.177748	-0.188962	-0.22179	-0.27548	-0.651699	-5.9773	-0.0181213	-0.050231	0.000754152
-0.257759	-0.260552	-0.312994	-0.395815	-1.14704	-0.222215	-0.316701	0.0771616	-0.0540029	-0.0656803