# Persistence in Convergence and Club Formation


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**Abstract**

In this paper, we examine the convergence hypothesis using a long memory framework that allows for structural breaks and does not rely on a benchmark country using both univariate and multivariate estimates of the long memory parameter $d$. Using per capita GDP gaps, we confirm the findings of non-stationarity and long memory behavior that have been found previously in the literature using univariate tests. However, the support for these findings is much weaker when using a multivariate framework, in which case we find more evidence of stationary behavior. Based on these results, we also investigate club formation, something that would suggest the presence of conditional convergence. We describe a club formation methodology using the sequential testing criteria that we have employed in our analysis as the basis for forming clusters or clubs of countries with similar convergence characteristics.



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# 1 Introduction

In a recent paper Stengos and Yazgan (2014a)[1] use a long memory analytical framework to examine the convergence hypothesis based on the estimation of $d$, the parameter that describes the underlying (long-memory) process and determines the speed of convergence of output (GDP per capita) gaps between different economies. The main finding of that paper is that although the long memory framework of analysis is much richer than a simple $I(1)/I(0)$ alternative, which produces a simple absolute divergence and rapid convergence dichotomy, the latter seems to be sufficient to capture the behavior of the gaps in per capita GDP levels and growth rates. The former produces a pattern of divergence whereas the latter produces a pattern of rapid convergence. Overall, it was found that any evidence of mean reversion and long memory was not strong enough, which is in contrast to some previous work in the literature that also uses a long-memory framework to analyze convergence, see Dufrénot et al. (2012). However, all previous research has relied on the univariate estimation of the long-memory parameter $d$, without accounting for possible correlations among the different output gaps (country differences), an issue that we want to address in this paper by employing a multivariate estimation and testing framework, following Shimotsu (2007). Using the latter methodology and in contrast to the results obtained using univariate estimation, we find evidence of mean reversion and slow (stationary) convergence. This evidence suggests that the overwhelming evidence in favor of divergence found in the literature may be partly explained by the use of methods that do not allow for interdependence among the persistence parameter. The latter acts as a moderating mechanism against divergence, as output pairs that appear to follow a non-stationary trajectory and non-convergence may individually be pulled back towards stationarity and convergence by their dependence on pairs that are stationary and convergent. This observation confirms the usefulness of multivariate long memory methods to address the issue of convergence, as they utilize more information than their univariate counterparts.

Using the results from the above analysis, we proceed to further examine the evidence of long memory type (absolute) convergence that we discovered. We proceed to investigate the possibility of club formation, a factor that would suggest the presence of conditional convergence. In that case, initial conditions would partly determine at least the long-run outcomes, and if

[1] see also Stengos and Yazgan (2014b)

countries with similar starting points exhibit similar long-run economic behavior, one could speak of convergence clubs. Club formation has recently become a very active area of research, as there are many different ways in which one can explore their presence (or absence). We will present a methodology on club formation based on the testing criteria that we have followed in our analysis thus far, and we will employ results from graphing theory to provide evidence for the existence of such clubs in our group of countries.

The remainder of the paper is organized as follows. The next section presents the methodology. We then proceed to present the data and results of the different tests that we apply to the per capita output gaps. We then proceed to a more detailed discussion of the methodology that we employ in the convergence club analysis. The final section concludes.

## 2 Testing framework with long memory.

The simple univariate pair-wise difference between the log of per capita income of country $i$ and $j$ at time $t$ is defined as

$$U_t = Y_t^i - Y_t^j = \beta(t) + Z_t \quad Z_t \sim I(d),\ i = 1, ..., N,\ i \neq j,\ t = 1, ..., T \tag{1}$$

The process $Z_t$ is described as $(1-L)^d Z_t = \varepsilon_t$, where $L$ is the lag operator and $\varepsilon_t$ is the disturbance term. The fractional integration parameter is given by $d$ under the assumption that the process is invertible ($d > -0.5$). The $\beta(t)$ function is a deterministic function of the time trend $t$ and can be linear, as in $\beta(t) = \beta_0 + \beta_1 t$. Alternately, as in Stengos and Yazgan (2014a), it can be defined in a way that admits structural breaks.

$$\beta(t) = \beta_0 + \beta_1 \sin\left(\frac{2\pi k t}{T}\right) + \beta_2 \cos\left(\frac{2\pi k t}{T}\right) \tag{2}$$

This functional form allows for the presence of (smooth) structural breaks. Note here that different values of $k$ will have different implications for the permanent or transitory nature of the breaks. If $k$ is an integer, temporary breaks will result, whereas fractional frequencies would imply permanent breaks because the function would not complete a full oscillation. One advantage of adopting this specification for structural breaks is that it does not require any prior knowledge of the dates on which those breaks occur. On the contrary, it assumes that breaks happen smoothly instead of abruptly, something that would make their detection more difficult.

In a multivariate setting, the long memory process underlying Equation (1) can be expressed as

$$\begin{pmatrix} (1-L)^{d_1} & & 0 \\ & \ddots & \\ 0 & & (1-L)^{d_q} \end{pmatrix} \begin{pmatrix} Z_{1,t} \\ \vdots \\ Z_{q,t} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{q,t} \end{pmatrix}, \qquad -\frac{1}{2} < d_1, ..., d_q < \frac{1}{2}\ , \tag{3}$$

where $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{qt})'$ is a covariance stationary process whose spectral density $f_\varepsilon(\omega_j)$ is bounded and bounded away from zero at zero frequency $\omega_j = 0$ (see Shimotsu (2007)). In the multivariate setting, the elements of the $q-$dimensional vector $\mathbf{Z}_t$ are interdependent and correlated with each other, in contrast with the univariate analysis.[2]

Following Stengos and Yazgan (2014a), one can distinguish between different convergence cases that are implied by different values of $d$. We follow that approach, which allows for a much richer classification of convergence types whereby one can distinguish between rapid convergence, stationary convergence and mean reverting non-stationary convergence, where initial differences either decay rapidly and play no role, or linger and have a lasting influence on the present, or fall somewhere in between them.

As in Stengos and Yazgan (2014a), we will concentrate on the estimated values of $d$ and provide tests of convergence based on these estimates. In the next section we will elaborate on the different testing strategies that we will adopt.

## 2.1 Testing for convergence.

We will consider two types of tests on the estimated $d$s. The first test is based on the estimation of $d$s using the multivariate approach illustrated in Equation (3). In this approach, the long memory parameters $\mathbf{d} = (d_1, ..., d_q)'$ are jointly estimated by the semiparametric estimator of Shimotsu

[2] As will be clear below in the present application, $q$ is equal to the number of pair-wise differences between the log of per capita incomes of the countries in our data set.

(2007), which uses only Fourier frequencies in the neighborhood of the origin. Let $I_Z(\omega_j)$ denote the periodogram of a series $Z_t$ based on a discrete Fourier transform $W_Z(\omega_j)$ at frequency $\omega_j = \frac{2\pi j}{T}$ for $j = 0, ...T-1$, such that $I_Z(\omega_j) = W_Z(\omega_j) W_Z^*(\omega_j)$ with $W_Z^*(\omega_j)$ are the complex conjugate of $W_Z(\omega_j)$ ,defined as $W_Z(\omega_j) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} Z_t e^{it\omega_j}$. As shown by (Shimotsu, 2007, p. 281), the periodogram $I_Z(\omega_j)$ can be used to define a multivariate estimator of $d$ obtained by minimizing an appropriate likelihood function. This estimator is asymptotically normally distributed with a variance-covariance matrix that is positively related to covariances among the long memory parameters $\mathbf{d} = (d_1, ..., d_q)^{'}$, i.e., variances are increasing with correlations among $\mathbf{d}$.

The second type of test is based on standard univariate estimation approaches that were used in Stengos and Yazgan (2014a). Based on the estimates of the $d$s (either multivariate or univariate), we perform the following tests on each of them

**Test 1:** $H_0^1 : d = 0$ against $H_A^1 : d > 0$ (rapid convergence against long memory)

**Test 2:** $H_0^2 : d = 0.5$ against $H_A^2 : d < 0.5$ (limit stationary long memory against stationary convergence)

**Test 3:** $H_0^3 : d = 0.5$ against $H_A^3 : d > 0.5$ "limit" stationary long memory against non-stationary mean reverting convergence)

**Test 4:** $H_0^4 : d = 1$ against $H_A^5 : d < 1$ (non-convergence against non-stationary mean reverting convergence)

**Test 5:** $H_0^5 : d = 1$ against $H_A^5 : d > 1$ (non-convergence against stochastic divergence)

We calculated critical values as described below and used them to perform the tests. Then, we compare the test results obtained by the multivariate estimator with those obtained with univariate estimators, as in Stengos and Yazgan (2014a). The univariate estimators covered in this paper include the Exact Local Whittle estimator of Shimotsu and Phillips (2005, 2006), Two Stage Feasible Exact Local Whittle of Shimotsu (2010), Fully Extended Local Whittle estimator of Abadir et al. (2007) and prior de-trending versions (see Shimotsu (2010)) of the 2 latter estimators. In all, we make use of four univariate and one multivariate test.

## 2.2 De-trending for structural breaks

To control for structural breaks, we "de-trend" data by estimating $\beta_0, \beta_1, \beta_2$, and $k$ in Equation (2) with the nonlinear least squares. Then, we subtract $\beta(t)$ function, estimated as such, from the data series $U_t$, before the estimation of $\mathbf{d}$ and applications of the tests.

## 2.3 Monte Carlo based critical values.

We conduct Monte Carlo simulations to compute the critical values of the statistic corresponding to each of the above tests under the null hypothesis under consideration. The test statistic is computed as

$$\frac{\sqrt{\upsilon}(\widehat{d_q} - d_0)}{\sigma(\widehat{d_q})} \tag{4}$$

where $\upsilon$ is the bandwidth parameter, $d_0$ is the value of $d$ under the null hypothesis, $\widehat{d_q}$ is the estimate of $d$, and $\sigma(\widehat{d_q})$ is its standard error defined in (Shimotsu, 2007, p. 283). For the simulations of the critical values, we consider 50,000 iterations. For each iteration, we generate a series from $Z_t = U_t \sim I(d)$ for different values of $d$ corresponding to the different null hypotheses listed above. In the simulations, we assume that the data is already de-trended. De-trending for structural breaks after estimating the $\beta(t)-$function avoids the problem of having to rely on specific values of the $\beta-$parameters to obtain critical values in the simulations. Hence, the test results will avoid possible misspecification due to the reliance on "incorrect" $\beta$ parameter values [3] In other words, we do not rely on a specific $\beta(t)-$function with particular parametric values of the $\beta-$ parameters to obtain the critical values of the various test statistics. As mentioned above, we de-trend the data by estimating $\beta_0, \beta_1, \beta_2$, and $k$ in Equation (2) using the non-linear least square method.

In Table 1 we provide critical values at the 5 and 10 percent significance levels for $T = 100$, 200, and 500, along with thoseof the univariate Whittle estimators reported in Stengos and Yazgan (2014a). These critical values are then used in the empirical analysis that follows.

Table 1

[3] Ashley and Patterson (2010) suggest isolating and separately examing both a local mean (i.e., a non-linear trend or the realization of a stochastic trend) and its deviations as a modelling strategy that would complement the estimation of a fractionally integrated model.

# 3 Data.

We update our Maddison data set that was used in Stengos and Yazgan (2014a), and also include data from the Penn World Tables (PWT) in our analysis as an additional source. The Maddison data consist of annual GDP per capita data covering the period from 1950 to 2010 for 141 countries[4] and PWT data of annual GDP per capita for the period from 1950 to 2011 for 74 countries. The country coverage of both data sets are illustrated in Table 2.[5] Hence our sample corresponds to $T = 60$ and $N = 141$ for Maddison and to $T = 61$ and $N = 74$ for PWT.

We first investigate the convergence of GDP per capita for all of the 141 and 74 countries taken together as a group and then separately as belonging to different groups from different continents (the Middle East and Central Asia, Europe, AsiaPacific, Sub-Saharan Africa and the Western Hemisphere, and for developing countries taken separately as a single group)[6]. These groups of countries are listed in Table 2 below.

Table 2

In addition to these geographical groups, we will also consider other categories based on levels of economic development, such as emerging markets, the Group of Seven (G7) and the OECD. Emerging markets are grouped according to both FTSE and S& P classifications. Moreover, we also use groupings based on data availability. Countries whose data are available from 1830, 1850, 1860, 1900, and 1930 onwards are taken as a group in the Maddison data set. These country groups are presented in Table 3 below.

Table 3

[4]The data come from the Maddison Project (Bolt and van Zanden, 2013). Some countries are missing observations at the end of the period in the final two years. The data are available at http://www.ggdc.net/maddison/maddison-project/home.htm, and they include all possible countries available.

[5]The PWT data come from Feenstra et al. (2013). We use PPP converted GDP per Capita G-K Methods in USD Dollars. Some countries have some missing observations at the beginning of the period, and 53 of these 74 countries are missing no observations. The remaining countries have some missing observations, but no country is missing more than 9. The data are available at http://www.rug.nl/research/ggdc/data/penn-world-table

[6]This classification is based on the usual classification made by the International Monetary Fund's regional economic outlook documents.

# 4 Empirical Findings.

Following Pesaran (2007), to analyze output per capita convergence across 141 and 74 countries, we apply the five tests discussed above (each corresponding to a convergence classification) to all possible pairs of $U_t = Y_t^i - Y_t^j, i = 1, 2, ..., N-1$, and $j = i+1, 2, ..., N$ in a sequential manner. Hence, we examine all $N(N-1)/2 = 9,870$ and $N(N-1)/2 = 2,701$ output gaps for Maddison and PWT deta sets, respectively. Under the null hypothesis of each test, we would expect the fraction of output gap pairs for which the null hypothesis is rejected to be close to the size of the test applied to the individual output gap pairs. Hence, in Table 4 below, rejection frequencies that greatly exceed a nominal size of 0.05 would be taken as evidence against the null. Conversely, rejection frequencies that are below the nominal size value will be taken as evidence in favor of the null[7].

The five tests are applied in sequential order in the sense that we continue to apply them until we find evidence in favour of some type of convergence, if there is any. The column denoted by ALL of Tables 4 and 5 summarizes the results of the four tests applied to all $9,870$ (Maddison) and $2,701$ (PWT) GDP per capita gap pairs at the 5 significance level based on critical values computed for $T = 100$. The table shows the rejection frequencies of the five tests defined above that are obtained from de-trended series using multivariate and univariate estimators, as described above.

Table 4 and Table 5

As shown in Table 4, all of the test results belonging to Test 1 report strong rejection of the null hypothesis of rapid convergence against the alternative of long memory. The evidence from Test 2, however, suggests that all of the tests calculated based on multivariate and univariate estimators find evidence in favor of the null hypothesis of a limit stationary long memory process. Test 3 registers very high rejection rates that conclusively indicate evidence in favor of limit stationary convergence and non-convergence. The fact that the rejection rates of Test 4, obtained from univariate estimators, are slightly above the 5 percent significance level, constitutes weak

[7]As shown by Pesaran (2007) Although the underlying individual tests are not cross-sectionally independent, under the null, the fraction of rejections is expected to converge to $\alpha$, as $N$ and $T \to \infty$, where $\alpha$ is the size of the underlying test.

evidence in favor of the alternative hypothesis of mean reverting non-stationary convergence, given possible size distortions due to the sequential nature of our testing procedure (although the different tests are assumed to be independent, there may still be size distortions).[8] However, the results associated with multivariate estimator of Shimotsu (2007) conclusively indicate mean reverting convergence with a much larger rejection rate. These results also hold for all of the grouping varieties considered in Table 4.

The evidence presented in Table 5, obtained from the smaller dataset of PWT, generally confirms the results obtained from the Maddison dataset, although some evidence on stationary convergence is visible when Test 2 and multivariate estimators are considered for the Western hemisphere in particular. The test based on multivariate estimators also shows that the European and Middle-East and Asian countries also display weak evidence of stationary convergence.[9]

We present the group results in Table 6 for the data from the Maddison data set only. Although the country groups whose data are available from 1830 and 1850 onwards provide evidence on stationary convergence, for the remaining groups, the evidence is somewhat weaker.

Table 6

The evidence is contrary to the previous findings in Stengos and Yazgan (2014a), where using only univariate statistics provided strong evidence in favor of (absolute) non-convergence, which is also confirmed here but not for the multivariate statistic. The difference in the evidence found using the latter as opposed to the former can be explained by the fact that correlations among the estimates of the $d$'s result in the standardized multivariate test statistics to account for the interdependence among the different pairs. In that case, the variances of high $d$'s are mitigated by the presence of negative covariances with other pairs that result in smaller variances overall for the test statistics. Hence, pairs that appear to suggest non-stationary behavior and non-convergence on their own may be pulled back towards stationarity and convergence by their dependence on pairs that are stationary and convergent. In that case, the test statistics of the multivariate test may take "larger" absolute values on average than their univariate counterparts. This evidence in favor of the convergence hypothesis is all the more remarkable in that it is obtained without

[8]The results obtained in Stengos and Yazgan (2014a) are similar but weaker with slightly smaller rejection ratios.

[9]Although we stopped the sequence of testing at Test 2, relying on this evidence for the Western hemisphere, we continued for the remaining two groups to guard against possible size distortions in these tests.

relying on a benchmark country and allowing for the presence of structural breaks. To summarize, contrary to previous evidence, such as in Stengos and Yazgan (2014a) and Dufrénot et al. (2012), which relied on univariate statistics, using a multivariate approach to estimate the long memory coefficients results in evidence that points towards a mean reverting process for per capita output gaps. These results hold for all different groups of countries, the Middle East and Central Asia, G7, S&P, FTSE, and OECD groups. For Europe and two small country groups whose data dated back to 1830 and 1850, considerable evidence on stationary convergence is present. For Asia and the Pacific, the Western Hemisphere and for three relatively small groups of countries whose data is available from 1860, 1900 and 1930, the evidence on stationary convergence is present but weaker, leaving mean reverting convergence as a second possibility.

It is worth noting that as in Stengos and Yazgan (2014a), the results are based on pair wise comparisons for all possible pairs within a group, as opposed to relying on a benchmark or group leader, as in Dufrénot et al. (2012). Using a benchmark results in differences in output gaps are to be expected, whereas these differences are smoothed out if gaps are only constructed as a difference of individual countries from the leader in the group. This is certainly true for the univariate tests, all of which point towards a long memory non-stationary behavior in the transitional dynamics of the output gaps. The evidence from the multivariate test, however, points towards mean reverting convergence irrespective of the absence of a benchmark leader country due to the greater interdependence between the different pairs captured by this test, which was ignored entirely by its univariate counterparts. We will now proceed to further analyze the evidence found above by exploring the possibility of conditional convergence and club formation.

# 5 Convergence Clubs

The above analysis largely implies that the dominant form convergence is of a non-stationary mean reverting nature, which seems to hold unconditionally for all countries. However, the analysis also indicates that stationary convergence, another stronger form of convergence, is also present in smaller group of countries forming convergence clubs. These clubs indicate the presence of conditional convergence. The results indicate conditional convergence because the differences among some groups of countries show high persistence that can only be corrected in the very long run, indicating the presence of cross-country structural heterogeneity. If initial conditions determine, at least partly, long-run outcomes, and countries with similar initial conditions exhibit similar long-run outcomes, then one can speak of convergence clubs (Durlauf et al. (2005)). This evidence on convergence clubs is provided on the basis of *a priori* defined group of countries or a group of countries dictated by data availability. In fact, this is the standard approach on which the literature on convergence clubs relies.[10] However, Phillips and Sul (2007) developed an algorithm that classifies groups endogenously rather than using *a priori* criteria. Similar to Phillips and Sul (2007), in this section, we also attempt to develop an endogenous clustering algorithm to determine the formation of convergence clubs using the above pair-wise framework. As stated by Pesaran (2007), "in principle, the convergence results from the analysis of pair-wise output gaps can be used to form "convergence clubs", but special care must be taken in addressing the specification search bias that such a strategy would entail." (Pesaran, 2007, p. 314)

Our approach attempts to pin down the convergence clubs for each type of convergence considered above. Among all pair-wise test results, we search for sets of countries that would yield the desired test result when subjected to the given test. For example, to obtain rapidly converging clubs, we search for sets of countries that would provide a rejection rate below 5 percent for Test 1 considered above. In other words, sets indicating non-rejection of the null of $H_0^1 : d = 0$ form rapidly converging clubs. To begin this search, we use all the pair-wise test results obtained from the all-country analysis. This problem can be solved by using an algorithm designed to find the maximal complete subgraph, or the maximal clique in graph theory terminology.

Consider the above four tests in terms of their implications for convergence types in terms of our sequential procedure. While non-rejection of the null of $H_0^1 : d = 0$ implies unconditionally rapid convergence, the non-rejection of $H_0^3 : d = 0.5$ implies the possibility of mean reverting convergence, provided that the non-rejection of the null of Test 2 has already been obtained in favor of limit stationarity. However, the rejection of the null of Test 2 and 4 provides evidence in favor of the alternatives, $H_A^2 : d < 0.5$ and $H_A^4 : d < 1$, implying stationary and mean reverting

[10] The issues of the definition of convergence clubs and their clustering have been widely discussed in the economic growth literature. Baumol (1986) has made the grouping on the basis of the countries' policy regimes (OECD member countries, centrally planned countries, and middle-income countries). Chatterji (1992) formed country sets based on their GDP per capita measures and applied cross-sectional econometric analysis. Hausmann et al. (2005), however, using a similar grouping approach, applied time series techniques. Another piece of evidence for the convergence club was suggested by Quah (1997), whose study showed that international income distribution, once unimodal, has been bimodal since 1960.

convergence. Denote the relevant hypothesis, either the null or alternative, by $\mathcal{H}^k$ to represent the convergence type under consideration. Furthermore, a set of countries $E$, forms a converging group in the sense implied by test $k$ if pairwise convergence holds for "all" $i$ and $j \in E,\ i \neq j$ at some allowed significance level. "The test" is applied to differences of all pairs of countries; we select the countries for which the test rejects the null (or alternatively, we select the countries for which the test does not reject the null). We explore all sets of countries in which all pairs reject (or all pairs do not reject) the null (apart from the trivial two element sets). In graph theory terms, pairwise test results form an undirected graph. Countries are vertices, and test results (rejecting or not rejecting pairs) determine the existence of edges. Thus, the problem is finding the maximal complete subgraph, or the maximal clique in graph theory terminology. The concepts are illustrated in the following figures (Figure 1 and 2).

Figure 1 and 2

Formally, let $U$ denote the set of all countries and $\mathcal{G}$ the class of all sets whose elements have the desired pairwise property such that

$$\mathcal{G} := \{E : \vee i, j \in E,\ \widehat{d}(Y_i - Y_j) \text{ passes (fails) certain test}\}$$

Then, the problem is as follows:

$$\arg\max_{\mathcal{G}}\{\ \#(E) :\ \ E \in \mathcal{G}\}$$

where $\#(\ )$ is the cardinality measure function on sets.

Finding the maximal cliques is a difficult problem in computational terms because its computational complexity is NP-complete such that solving it with brute force requires $2^N - \binom{N}{2} - N - 1$ trials. First, Bron and Kerbosch (1973) developed an algorithm that solves the problem in exponential time. Recently, several planar graph algorithms have been devised that solve the maximal clique problem in polynomial time. In our application, we employ Konc and Janezic (2007), which is an improved branch-and-bound algorithm that ends in polynomial time.

To illustrate our approach, we applied our procedure to the results we have already obtained. Because the approach requires a considerably large amount of computational time, we restrict our universe $U$ to a pre-selected group. Also note that as will be clear below, this does not place any restrictions on the main message presented below. The pre-selected groups are illustrated in Table 7, which presents the results of the search for different type of convergences.

Table 7

In Table 7, in the search for convergence clubs, when the type of convergence depends on the non-rejection of the null hypothesis (rapid convergence), we took a 10 percent rejection rate as the benchmark (we hereby attempt to compensate for possible over-rejection displayed by the different tests that we use). In other words, any club producing a 10 percent rejection rate or less is taken as evidence for the validity of the null of rapid convergence. However, we set a 50 percent benchmark rejection rate for the cases where the type of convergence depends on the rejection of the null hypothesis (stationary and mean reverting convergence), accounting for all the possible size distortions due to the sequential nature of our testing procedure.

For the case of rapid convergence, there is no evidence for convergence clubs because the set of 7 and 6 convergent pairs in Europe or in the G7 extended by the group of emerging countries (represented by S&P) are not able to form a convergence club with at least three countries. For the same country groups, 2 convergence pairs are able to form a convergent club of 3 in the case of stationary convergence. However, as shown in Table 7, there are many convergence clubs with different numbers of countries for the case of mean reverting convergence. Note also that these clubs are not required to be disjoint sets. The maximum size of these clubs is 12, and there are 6 different convergent clubs, each containing 12 countries. Figure 3 below illustrates one of these clubs with its associated universe.

Figure 3

We repeat the same analysis using the country groups based data availability as universes in which to search for convergence clubs. The results are illustrated in Table 8. We observe a similar pattern here. While there is little evidence for convergence clubs in rapid convergence, there is ample evidence for convergent clubs in the case of stationary convergence. We also provide a sample from rapidly converging clubs in Figure 4.

Table 8 and Figure 4

# 6 Conclusions.

In this paper, we use a long memory framework of analysis that does not rely on a benchmark country but allows for the presence of structural breaks to estimate the time series properties of output gaps for counties in the post-World War II period and, as such, provide evidence on the convergence hypothesis. The focus of the paper is first the estimation of $d$, the parameter that determines the speed of convergence between different economies. We estimate $d$ using a multivariate estimator from Shimotsu (2007) and a number of other univariate methods found in recent studies. The main finding of our paper is that for per capita GDP gaps, the parameter $d$ takes values in the range from $0.5 < d < 1$ (mean reverting convergence), and the range from $0 < d < 0.5$ (stationary convergence) seems be a possibility, especially for some country groups when using the multivariate approach.

The difference between univariate and multivariate procedures in these findings can be explained by the fact that correlations among the estimates of the $d's$ in the multivariate test statistics account for the interdependence among the different pairs. In that case, any divergent behavior is mitigated by the presence of convergent pairs that act as stabilizing factors for the group as a whole.

Using the results from the above analysis, we proceed to further examine the evidence we found for long memory type (absolute) convergence. We then investigate the possibility of club formation, something that would suggest the presence of conditional convergence, and offer a club formation methodology using the sequential testing criteria that we have employed in our analysis as the basis of club formation.

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# Tables

Table 1: Empirical critical values of Test1, 2, 3, and 4 for T = 100, 200, and 500.

| | Test 1 | | | Test 2 | | | Test 3 | | | Test 4 | | | Test 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CV | 95% | | | 05% | | | 95% | | | 05% | | | 95% | | |
| T | 100 | 200 | 500 | 100 | 200 | 500 | 100 | 200 | 500 | 100 | 200 | 500 | 100 | 200 | 500 |
| FELW | 2.207 | 1.990 | 1.812 | -2.216 | -2.076 | -1.930 | 2.254 | 2.071 | 2.084 | -2.253 | -2.153 | -2.000 | 2.180 | 1.974 | 1.798 |
| FELWd | 1.945 | 1.765 | 1.624 | -3.079 | -2.668 | -2.355 | 2.149 | 1.939 | 1.938 | -2.647 | -2.235 | -2.042 | 2.175 | 1.975 | 1.795 |
| 2FELW | 2.206 | 1.990 | 1.812 | -2.216 | -2.076 | -1.930 | 2.253 | 2.018 | 1.795 | -2.312 | -2.153 | -2.000 | 2.180 | 1.974 | 1.798 |
| 2FELWd | 1.944 | 1.765 | 1.624 | -3.079 | -2.668 | -2.354 | 2.153 | 1.878 | 1.670 | -2.534 | -2.235 | -2.042 | 2.175 | 1.975 | 1.795 |
| MLW | 2.02 | 1.985 | 1.924 | -2.879 | -2.268 | -2.082 | 2.013 | 1.912 | 1.785 | -2.267 | -2.125 | -1.942 | 2.004 | 1.854 | 1.536 |

Notes: FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out assuming $\upsilon = T^{0.6}$ for all Whittle estimators.

Table 2: Countries and group of countries belonging to Maddison and PWT datasets.

| | **Maddison and PWT** | **Only PWT** | **Only Maddison** |
|---|---|---|---|
| Middle-East and Central Asia | Egypt, Iran, Islamic Republic of, Jordan, Morocco, Pakistan | | Afghanistan, Bahrain, Iraq, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syrian Arab Republic (Syria), United Arab Emirates, Yemen, Palestinian Territory, Occupied, Algeria, Djibouti, Libya, Mauritania, Somalia, Sudan, Tunisia |
| Europe | Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Sweden, Turkey | Cyprus, Iceland, Luxembourg, Malta | Albania, Bulgaria, Czechoslovakia, Hungary, Poland, Romania, Yugoslavia, Croatia, Macedonia, Republic of, Slovenia |
| Asia and Pacific | Australia, Bangladesh, China, India, Japan, Korea, Republic of, Sri Lanka, Malaysia, New Zealand, Philippines, Thailand, Taiwan, Republic of China | | Indonesia, Myanmar, Hong Kong, Special Administrative Region of China, Nepal, Singapore, Cambodia, Lao PDR, Mongolia, Korea, Democratic People's Republic of, Viet Nam |
| Sub-Saharan Africa | Benin, Burkina Faso, Congo, Democratic Republic of the, Ethiopia, Ghana, Guinea, Kenya, Mauritius, Malawi, Nigeria, Uganda, South Africa, Zambia | Zimbabwe | Angola, Botswana, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo (Brazzaville), Côte d'Ivoire, Equatorial Guinea, Gabon, Gambia, Guinea-Bissau, Lesotho, Liberia, Madagascar, Mali, Mozambique, Namibia, Niger, Rwanda, Sao Tome and Principe, Senegal, Seychelles, Sierra Leone, Swaziland, Tanzania, United Republic of, Togo |
| Western Hemisphere | Argentina, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, Guatemala, Honduras, Jamaica, Mexico, Panama, Peru, Paraguay, El Salvador, Trinidad and Tobago, Uruguay, United States of America, Venezuela (Bolivarian Republic of) | | Cuba, Haiti, Nicaragua, Puerto Rico |

Table 3: Country Groups based on Economic Characteristics and Data Availability

| | |
|---|---|
| 1830 | Italy, Sweden, UK, USA, Denmark, France, Netherlands, Norway, Australia |
| 1850 | 1830 + Belgium, Germany, Greece, Spain |
| 1860 | 1850 + Finland, Switzerland |
| 1900 | 1860 + Austria, Portugal, New Zealand, Canada, Brazil, Chile, Colombia, Peru, Uruguay, Venezuela, Japan, Sri Lanka, Argentina, Mexico, Ecuador, India |
| 1930 | 1900 + Ireland, Turkey, Costa Rica, Guatemala, South Africa |
| G7 | France, Germany, Italy, UK, Canada, USA, Japan |
| FTSE | Hungary, Poland, Brazil, Mexico, Thailand, Taiwan, Malaysia, Turkey, South Africa |
| S&P | Brazil, Hungary, Poland, Chile, Colombia, Mexico, Peru, China, India, Philippines, Thailand, Taiwan, Malaysia, Turkey, Egypt, Morocco, South Africa |
| OECD | Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, UK, Ireland, Greece, Portugal, Spain, Australia, New Zealand, Canada, USA, Hungary, Poland, Chile, Mexico, Japan, South Korea, Singapore, Israel, Turkey |

Table 4: Rejection frequencies of Test 1, Test 2, Test 3, and Test 4 for Maddison Data.

| | | ALL | EUR | WHE | MEA | AAP | SSA | G7 | S&P | FTSE | OECD |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Test 1 | FELW | 0.998 | 1.000 | 0.997 | 0.996 | 1.000 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | FELWd | 0.998 | 1.000 | 0.997 | 0.996 | 1.000 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELW | 0.998 | 1.000 | 0.997 | 0.996 | 1.000 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELWd | 0.998 | 1.000 | 0.997 | 0.996 | 1.000 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| | MLW | 0.996 | 1.000 | 1.000 | 1.000 | 0.978 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| Test 2 | FELW | 0.015 | 0.011 | 0.007 | 0.018 | 0.039 | 0.017 | 0.000 | 0.000 | 0.000 | 0.045 |
| | FELWd | 0.014 | 0.011 | 0.007 | 0.014 | 0.035 | 0.014 | 0.000 | 0.000 | 0.000 | 0.045 |
| | 2FELW | 0.014 | 0.011 | 0.013 | 0.014 | 0.039 | 0.016 | 0.000 | 0.000 | 0.000 | 0.045 |
| | 2FELWd | 0.013 | 0.011 | 0.013 | 0.011 | 0.035 | 0.013 | 0.000 | 0.000 | 0.000 | 0.045 |
| | MLW | 0.034 | 0.032 | 0.007 | 0.014 | 0.056 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 |
| Test 3 | FELW | 0.974 | 0.981 | 0.993 | 0.971 | 0.944 | 0.973 | 0.857 | 1.000 | 1.000 | 0.955 |
| | FELWd | 0.974 | 0.981 | 0.993 | 0.967 | 0.944 | 0.974 | 0.857 | 1.000 | 1.000 | 0.955 |
| | 2FELW | 0.974 | 0.979 | 0.987 | 0.975 | 0.944 | 0.973 | 0.857 | 1.000 | 1.000 | 0.955 |
| | 2FELWd | 0.975 | 0.979 | 0.987 | 0.975 | 0.944 | 0.974 | 0.857 | 1.000 | 1.000 | 0.955 |
| | MLW | 0.903 | 0.921 | 0.940 | 0.899 | 0.905 | 0.950 | 1.000 | 0.938 | 0.875 | 1.000 |
| Test 4 | FELW | 0.140 | 0.119 | 0.100 | 0.174 | 0.121 | 0.121 | 0.286 | 0.188 | 0.375 | 0.273 |
| | FELWd | 0.132 | 0.108 | 0.090 | 0.174 | 0.117 | 0.114 | 0.286 | 0.188 | 0.375 | 0.227 |
| | 2FELW | 0.140 | 0.122 | 0.097 | 0.174 | 0.117 | 0.121 | 0.286 | 0.188 | 0.375 | 0.273 |
| | 2FELWd | 0.135 | 0.116 | 0.093 | 0.174 | 0.117 | 0.115 | 0.286 | 0.188 | 0.375 | 0.227 |
| | MLW | 0.473 | 0.418 | 0.503 | 0.576 | 0.468 | 0.362 | 0.286 | 0.625 | 0.750 | 0.409 |

Notes: The abbreviations used in the table are as follows: ALL (All countries), AAP (Asian and Pacific countries), MEA (Middle-East and Asian countries), EUR (European countries), SSA (Sub-Saharan countries), WHE (Western-hemisphere countries), G7: Group of 7 countries, OECD: OECD countries, FTSE: Financial Times emerging market country group,S&P: Standart and Poors country group, FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $\upsilon = T^{0.6}$ for all Whittle estimators..

Table 5: Rejection frequencies of Tests 1, 2, 3, and 4 for PWT data.

| | | ALL | EUR | WHE | MEA | AAP | SSA | G7 | S&P | FTSE | OECD |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Test 1 | FELW | 0.995 | 0.996 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| | FELWd | 0.995 | 0.996 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELW | 0.996 | 0.996 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELWd | 0.996 | 0.996 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| | MLW | 0.973 | 0.926 | 1.000 | 0.970 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Test 2 | FELW | 0.024 | 0.035 | 0.000 | 0.061 | 0.022 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 |
| | FELWd | 0.024 | 0.035 | 0.000 | 0.045 | 0.022 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2FELW | 0.023 | 0.035 | 0.000 | 0.061 | 0.022 | 0.019 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 2FELWd | 0.023 | 0.035 | 0.000 | 0.045 | 0.022 | 0.014 | 0.000 | 0.000 | 0.000 | 0.000 |
| | MLW | 0.076 | 0.108 | 0.200 | 0.106 | 0.011 | 0.052 | 0.000 | 0.000 | 0.000 | 0.000 |
| Test 3 | FELW | 0.937 | 0.931 | - | 0.864 | 0.945 | 0.919 | 1.000 | 1.000 | 1.000 | 1.000 |
| | FELWd | 0.941 | 0.939 | - | 0.864 | 0.934 | 0.924 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELW | 0.940 | 0.935 | - | 0.864 | 0.945 | 0.933 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 2FELWd | 0.942 | 0.939 | - | 0.864 | 0.934 | 0.933 | 1.000 | 1.000 | 1.000 | 1.000 |
| | MLW | 0.807 | 0.814 | - | 0.758 | 0.868 | 0.810 | 0.800 | 0.667 | 0.750 | 0.810 |
| Test 4 | FELW | 0.216 | 0.216 | - | 0.288 | 0.187 | 0.195 | 0.000 | 0.250 | 0.250 | 0.190 |
| | FELWd | 0.207 | 0.212 | - | 0.258 | 0.176 | 0.176 | 0.000 | 0.250 | 0.250 | 0.190 |
| | 2FELW | 0.212 | 0.203 | - | 0.273 | 0.187 | 0.186 | 0.000 | 0.333 | 0.500 | 0.238 |
| | 2FELWd | 0.209 | 0.203 | - | 0.258 | 0.187 | 0.181 | 0.000 | 0.333 | 0.500 | 0.238 |
| | MLW | 0.721 | 0.714 | - | 0.773 | 0.868 | 0.733 | 0.800 | 0.750 | 0.750 | 0.810 |

Notes: The abbreviations used in the table are as follows: ALL (All countries), AAP (Asian and Pacific countries), MEA (Middle-East and Asian countries), EUR (European countries), SSA (Sub-Saharan countries), WHE (Western-hemisphere countries), G7: Group of 7 countries, OECD: OECD countries, FTSE: Financial Times emerging market country group,S&P: Standart and Poors country group, FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $\upsilon = T^{0.6}$ for all Whittle estimators.

Table 6: Rejection frequencies of Tests 1 and 2 for group of countries having available data since some selected years between 1830 and 1930 according to Maddison's data.

| | | 1830 | 1850 | 1860 | 1900 | 1930 |
|---|---|---|---|---|---|---|
| Test 1 | FELW | 0.916 | 0.846 | 0.876 | 0.940 | 0.946 |
| | FELWd | 0.972 | 0.846 | 0.876 | 0.942 | 0.949 |
| | 2FELW | 1.000 | 0.846 | 0.876 | 0.940 | 0.946 |
| | 2FELWd | 1.000 | 0.846 | 0.876 | 0.942 | 0.950 |
| | MLW | 1.000 | 0.987 | 0.952 | 0.953 | 0.954 |
| Test 2 | FELW | 0.222 | 0.372 | 0.210 | 0.120 | 0.127 |
| | FELWd | 0.222 | 0.372 | 0.181 | 0.110 | 0.121 |
| | 2FELW | 0.111 | 0.308 | 0.210 | 0.120 | 0.135 |
| | 2FELWd | 0.111 | 0.308 | 0.181 | 0.110 | 0.124 |
| | MLW | 0.222 | 0.294 | 0.133 | 0.140 | 0.141 |

Notes: FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $\upsilon = T^{0.6}$ for all Whittle estimators.

Table 7: Convergence Clubs

| Test Type | Country Groups | Club Sizes | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | # 2 | # 3 | # 4 | # 5 | # 6 | # 7 | # 8 | # 9 | # 10 | # 11 | # 12 |
| Rapid Conv. ($H_0^1 : d = 0$) | G7 + Europe | 1 | - | - | - | - | - | - | - | - | - | - |
| | Europe + Emerging | 7 | - | - | - | - | - | - | - | - | - | - |
| | G7 + Emerging | 6 | - | - | - | - | - | - | - | - | - | - |
| Stationary Conv. ($H_A^2 : d < 0.5$) | G7 + Europe | - | - | - | - | - | - | - | - | - | - | - |
| | Europe + Emerging | 2 | 1 | - | - | - | - | - | - | - | - | - |
| | G7 + Emerging | 2 | 1 | - | - | - | - | - | - | - | - | - |
| Mean Reverting Conv. ($H_A^4 : d < 1$) | G7 + Europe | 38 | 726 | 3246 | 3737 | 4577 | 6185 | 3639 | 731 | 67 | - | - |
| | Europe + Emerging | 96 | 2971 | 18474 | 25220 | 41378 | 84168 | 72565 | 24080 | 6925 | 1397 | 6 |
| | G7 + Emerging | 66 | 310 | 1739 | 3353 | 2533 | 2586 | 3265 | 1605 | 243 | 23 | - |

Table 8: Convergence Clubs

| Club Type | Years | Club Sizes | | | |
|---|---|---|---|---|---|
| | | # 2 | # 3 | # 4 | # 5 |
| Rapid Conv. | 1830 | - | - | - | - |
| | 1850 | 12 | 2 | - | - |
| | 1860 | 14 | 2 | - | - |
| | 1900 | 29 | 3 | - | - |
| | 1930 | 37 | 3 | - | - |
| Stationary Conv. | 1830 | - | - | - | - |
| | 1850 | 12 | 23 | 49 | 25 |
| | 1860 | 14 | 30 | 72 | 28 |
| | 1900 | 29 | 90 | 381 | 208 |
| | 1930 | 38 | 139 | 649 | 299 |

Figure 1: A sample undirected graph

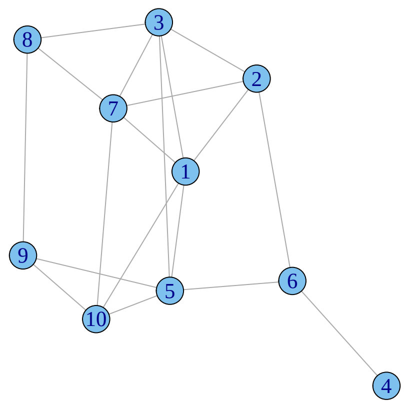


Figure 2: A sample maximum clique

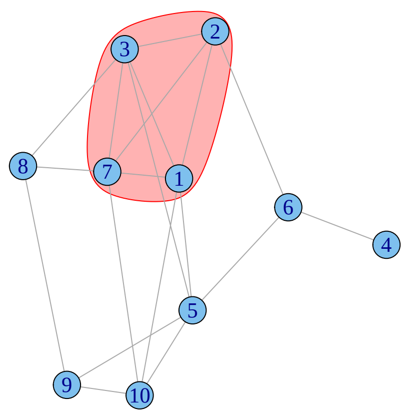

Figure 3: A Club of Mean Reverting Convergence (Europe + Emerging Markets)

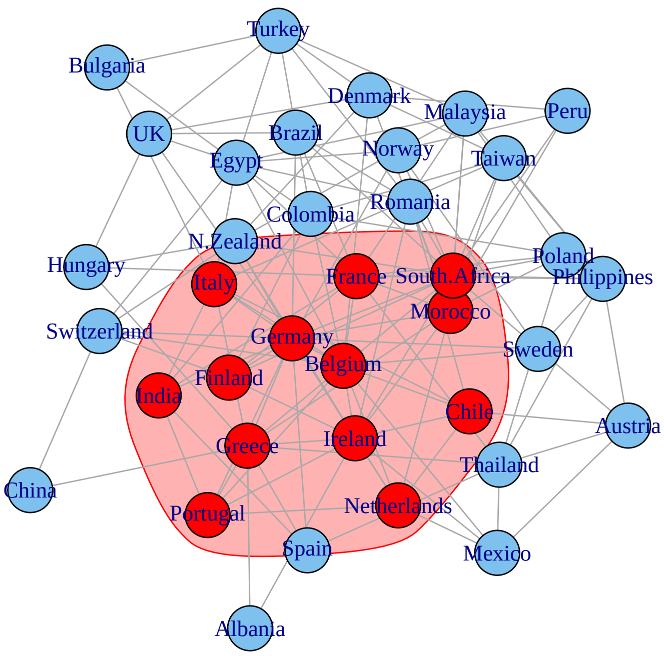

Figure 4: A Rapidly Converging Club (1930)