The Embodiment of Intangible Investment Goods: a Q-Theory Approach*

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Abstract

Recent empirical findings on firms’ expenditure towards the creation and acquisition of knowledge goods, otherwise known as intangibles, suggest that their share in overall investment has grown considerably. Still, intangible investment is rarely present in investment models. This paper extends the q-theory of investment to include intangible investment and uses the model to measure the contribution of intangible goods to overall capital stock in the U.S. The model highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment to overall capital stock. In particular, given that the relative cost of the main input to intangible production, skilled labor, rose substantially in the 80s and 90s, the price of intangibles inherits this rise. As a result, the downward trend in the aggregate investment deflator series reported by national accounts, which accounts only for the presence of tangible investment goods, is found to have a significant downward bias in the 90s. The model also shows that the growth in overall capital stock from the late-80s until 2000 was mainly driven by an increase in the contribution of intangibles. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the price movement of intangibles beyond only their rising share in overall investment.

Keywords: Intangible investment, Q-Theory, Skill Premium, Investment Deflator.

JEL classifications: E22, E44, O47, G31, J31, J44.

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1 Introduction

Recent empirical findings point to a considerable increase in the share of intangible expenditures in overall investment since the early 80s. Still, intangible investment is rarely accounted for in investment models. This paper extends the q-theory of investment to account for both the existence of intangible investment goods and the movement of their price over time. The model highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment good to overall capital stock. In particular, the model is used to measure the contribution of embodied intangible investment goods to overall capital stock in the U.S.

The increasing share of intangibles since 1950 is documented in a comprehensive study by Corrado et al. (2006). The authors construct a data set on U.S. firms' spending on an identified list of intangible inputs\(^1\). This list consists of three main categories: computerized information (software and database expenses), innovative property (scientific and non-scientific R&D) and economic competencies (training expenses, advertising expenditures and spending on organizational change and design)\(^2\). They find that the share of intangible investment in overall investment since 1950 has steadily increased over time and that by 2000, investment in intangible capital was as large as investment in physical capital\(^3\).

Other authors attempted to measure the size of intangibles by taking indirect approaches. For example, Hall (2001) uses a q-theory model to infer the amount of intangible capital stock such that his model matches the movements in the aggregate value of securities. McGrattan and Prescott (2007a) use a general equilibrium model to generate the intangible series which makes movements in hours worked in producing intangibles, otherwise unaccounted for, match some aggregate moments.

A key limitation to all these approaches is that none of them separately identify indices for the price and quantity of intangible investment goods. Corrado et al. (2006) and Hall (2001) simply assume that the price of intangibles is the same as the price of tangible investment.

\(^1\)A consensus emerged over time among national accountants on what those items should be. See Vosselman (1998).

\(^2\)See the appendix in Belhocine (2007b) for a list and a description of specific items included in each group.

\(^3\)Similar work conducted for Canada, the UK and Japan respectively by Belhocine (2007b), Marrano and Haskel (2006) and Fukao et al. (2007) reached similar findings.
goods. McGrattan and Prescott (2007a) guess a price series in order to infer an intangible investment series.

The distinction between the price and the quantity of intangibles is an important issue because it is not clear how much of the increase in the overall expenditure on intangibles is attributable to the real contribution of intangibles. Moreover, Hall’s lack of distinction between the price of tangibles and intangibles effectively implies some unlikely movements in the stock of intangibles, including that they were negative for a decade after 1974 and extremely volatile in the last decade.

What is more, the substantial increase in the relative cost of the main input to intangibles production, skilled labor, suggests that the relative price of intangibles also rose (Katz and Autor (1999) and Lemieux (2007) review the literature on the rise in the skill-premium). A closer look at the aforementioned list of intangible items reveals that they are mainly produced by a class of skilled workers made of university graduates and executives. The compensation of these two types of workers has been increasing since the early 1980s until it peaked in 2000 and declined. This compensation pattern should translate into a rise in the unit cost of intangibles up to the year 2000 and a subsequent fall.

In order to account for both the existence of intangible investment goods and the movement of their price over time, I develop and implement a generalized q-theory of investment. Previous authors have mainly assumed that intangible goods are disembodied and that the stock of intangibles can be conceptualized as separately evolving from the physical capital stock of firms. However, most intangible investments made by firms appear to be “embodied” in a composite stock made of tangible and intangible capital. One might think of this composite stock as combinations of “hardware” (e.g. office buildings, computers, machines) and “software” (organizational design, operating systems, blueprints) which have been brought together by past investments.

This is precisely how the investment process is modeled, with units of intangible and tangible goods being optimally combined as intermediates in the production of a final aggregate investment good. It is this final investment good that is accumulated overtime and used in output production. Changes in the share of expenditures on the two types of investment goods over time reflect exogenous technological change in the final investment good sector.
The price of intangible goods reflects the costs of producing them which, in turn, reflects the wages of skilled labor. I construct a price series for intangibles based upon the compensation received by university graduates and executives and estimates of their share in the production of intangibles (similar to those used by Corrado et al. (2006)).

Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts exhibits a downward trend starting in the mid-1950s, the investment deflator constructed from the model to include intangible investment has a downward trend up to the mid-1980s and then rises up to 2000 and declines afterwards\(^4\). In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for.

The model successfully generates a smoothly-behaving series of capital stock with a market value that predominantly remains above and close to its acquisition cost; this reflects a Tobin’s q that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin’s q when capital is exclusively made of tangible investment. This result is a direct implication of: 1) the new trend observed in the acquisition cost of capital once intangibles are accounted for and 2) the increased size of capital stock once intangible investment goods are accounted for. These successful quantitative implications suggest that the mixed econometric success of the q-theory might be a consequence of the omission of intangible investment\(^5\).

Finally, the model is used to measure the extent to which the composition of the stock of capital in the economy has shifted over time towards the inclusion of more intangible capital at the expense of tangible capital. The decomposition of capital stock into its investment constituents shows that the rise in overall capital stock from the mid-80s until the late 90s was mainly driven by an increase in intangible investment despite an increase in tangible investment. In particular, I find that once the price movements of the two investment goods

\(^4\)The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood et al. (1997) and Krusell (1998).

\(^5\)These potential specification problems are mentioned in a related discussion by Hall (2004) pp. 914-915. See the conclusion section at the end of this paper for an elaboration on this point.
are accounted for, the relative contribution of accumulated intangible investments to overall capital stock was higher prior the 1990s and substantially lower during the 1990s than other authors have found.

Taken together, the findings in this paper underscore the importance of intangible investment as a source of value for the firm and as a key component of any investment theory. Moreover, this paper provides a consistent account of the compositional changes that have occurred in the last 25 years in the U.S. economy by bringing together the evidence on the rise of the skill premium, the evidence on the increasing importance of intangible investment and the evidence on the behavior of aggregate securities.

The paper proceeds as follows. In section 2, the approach followed is contrasted with other approaches that try to account for the presence of intangibles. The most notable feature that distinguishes this work are: 1) the linkage of the unit cost of intangibles to the wage behavior of skilled workers and 2) the embodiment of intangibles in tangible investment goods. Section 3 outlines how the extension of the q-theory is constructed. It will also describe the approach used to disentangle the contribution of each type of investment good to capital stock. Section 4 discusses how the parameters are calibrated and what data sources are used to document some of these choices. This section also details the technology by which skilled workers, identified as university graduates and executives, produce intangibles. Section 5 describes the findings with regards to the inferred capital stock, the behavior of Tobins’ q and the changing composition of the capital stock over time. Section 6 compares the findings with the work and results of the two closest approaches in the literature, namely Hall (2001) and McGrattan and Prescott (2007a). Section 7 concludes with a discussion of future research.

2 Difference with Other Approaches

The baseline neoclassical model of investment, also known as the q-theory of investment, predicts that the decision of a firm to invest is a function of a trade-off between the benefit of increasing capital by one unit and the cost of acquiring and installing the extra unit. Hayashi (1982) showed the conditions under which the marginal benefit of increasing capital is identified with the net value of outstanding securities. Figure 1 shows the evolution of the aggregate value of net securities together with the acquisition cost of capital or its
replacement cost. The q-theory states that, from this graph, the firm can deduce the benefits and costs it faces in its decision to adjust capital by one unit: if the capital in place has a higher value than the capital that is not yet installed then the firm should take advantage of the arbitrage opportunity. Any discrepancy between the cost and benefit is due to capital adjustment costs which slow down the arbitrage process.

Another way of presenting this tradeoff is through Tobin’s q, which is the ratio of the net value of securities to the acquisition cost of capital. Tobin’s q reflects the incentive of the firm to adjust its capital stock: the firm should invest if the value is bigger than one, otherwise the firm should disinvest. Figure 2 illustrates the behavior of Tobin’s q.

Two aspects of this behavior are difficult to rationalize from the perspective of the q-theory. First, we observe a large discrepancy between the benefit and the cost of adjusting capital by one unit suggesting a very long adjustment period. In fact, the econometrics implementation of the q-theory by Summers (1981) implied an adjustment period of 8 years. This number is viewed as unrealistic. Second, the period from the mid-1970s until the mid-1980s features an aggregate value of firms below the acquisition cost of capital. This is

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6Other studies confirmed the long implied adjustment period. See Chirinko (1993) for a review of the early literature.
Three broad explanations have been proposed to explain the apparent failure of the q-theory to explain these anomalies in Tobin’s q. The first line of attack was to study how financial markets do not accurately reflect the fundamental value of the firm (see LeRoy (2004) for a review of this literature). The second line of attack was to relax the restrictive assumptions that allowed the value of the installed capital to reflect the marginal value of increasing the capital stock; the reliance on perfect competition, constant returns to scale and quadratic adjustment costs might not be accurate. (see Caballero (1999) for a review of this literature). Finally, one explanation that has received increasing attention in the past few years is the existence of another category of investment goods that has not yet been accounted for in theoretical and empirical investigations. This category of investment goods is made of intangibles. The argument follows that if the net value of securities reflects, under rational valuation, the value of installed capital stock, then periods of marked departure of the net value of securities from the acquisition cost of tangible capital stock is evidence for the accumulation of intangible capital by firms.

This last possibility was proposed by Hall (2001). His paper uses the q-theory of investment to generate the stock of capital in the economy. From this inferred capital, the component which is recorded in national income accounts as physical capital is subtracted
and the residual is assimilated to intangible capital. One crucial assumption in Hall’s work is that intangible and tangible investment goods are perfect substitutes and that their price is equal. These two assumptions are violated in the data as discussed in the introduction: the share of intangibles in overall capital expenditures has been increasing overtime and the unit cost of intangibles behaves the opposite of the price of physical capital. Finally, one anomaly in Hall’s findings is that the quantity of intangibles falls below zero for about a decade starting in the mid-70s.

Eliades and Weeken (2004) apply Hall’s methodology to the UK. These authors also find negative intangibles throughout the 70s, the 80s and early 90s. However, they reach the same qualitative results as Hall (2001) for the late 90s. Belhocine (2007a) applies this approach to Canada for the period after 1990 and reaches the same qualitative conclusions as those found for the U.S. and the U.K.

In this paper, I depart from Hall’s approach by: 1) relaxing the assumption of equality between the price of intangibles and tangibles and 2) allowing some degree of substitutability between the two investment goods.

McGrattan and Prescott (2005a) make another attempt to use the unmeasured levels of intangible capital to rationalize the rise in the US and UK stock markets in late 90s. The authors depart from Hall’s work by taking a general equilibrium approach with no frictions aside from the existence of taxes. The authors rationalize the size of intangible investment found in Corrado et al. (2005) in the 1990s while using the change in tax regulations to account for the differing performance between the UK and the US stock markets. McGrattan and Prescott (2005b) show that by explicitly accounting for intangible investment in an otherwise standard real business cycle model, one can explain the low productivity levels in the early 90s. In particular, they argue that GDP in national income accounts is undervalued because intangible investment is expensed, which ultimately created a downward bias in the productivity estimates for the early 90s. Taking this work further, McGrattan and Prescott (2007a) extend the baseline real business cycle model to allow for the production of intangibles by the representative agent. The goal of their work is to reconcile the real business cycle model’s prediction of a fall in hours worked after the 1990s with actual evidence of their increase. In particular, their model is calibrated to match some
aggregate macroeconomic series and features two stocks of capital: a tangible capital stock and an intangible capital stock.

In this paper, I calibrate the extended q-theory model to the aggregate value of securities and the extended model features only one homogeneous capital stock that embodies both tangible and intangible investment goods. Another difference with the work of McGrattan and Prescott is the way the price of intangibles is calculated: McGrattan and Prescott (2007b) state that they guessed the price of intangibles in their solution to derive a series of intangible investment. As mentioned above, I derived the price of intangibles from the behavior of the cost of its main inputs.

Finally, the proposed model is most comparable to the work of Hall (2000). In this paper, Hall focuses on the period from 1990 to 2000 and tries to link the behavior of university graduate wages with the formation of intangible capital. Hall does not however account for the changing structure of firms’ investment as done in the extended model I propose. In addition, Hall assumes that the intangible and tangible capital stocks evolve over time separately while I develop a model with a homogeneous capital stock. Moreover, the inclusion of executives in the class of skilled labor are another conceptual and empirical difference between Hall’s work and the model developed below. Finally, the long-term approach I adopt illuminate the pre-1990 events as well as the post-2000 events.

Although most studies find that the size of intangible capital is substantial, Hall (2003) and Bond and Cummins (2000) are exceptions. They both show, using different data, that the returns to physical capital exhaust all payments to capital and hence, nothing is leftover to reward the services of intangible capital. This is held as evidence for the absence of a substantial intangible capital stock which is puzzling in light of the findings in the previously cited papers. The conclusion section at the end of this paper will attempt to reconcile the findings of Hall (2003) and Bond and Cummins (2000) with the results obtained here.

3 Including Intangibles in the Q-theory of Investment

The standard neoclassical model of investment as developed in Hayashi (1982) is extended to account for the production of intangibles. Once produced, intangible investment goods are combined with tangible investment goods, which are bought from the market, to produce
a final investment good that accumulates into a capital stock which is used in production. Ultimately, the model relates the value of securities to the value of the capital in place within firms. This key relationship allows the generation of a series for the capital stock and the construction of a series for Tobin’s q.

There is perfect competition in input and output markets. The firm employs two types of labor, skilled and unskilled. An amount $l^u$ of unskilled labor is used for the production of output only. It is paid $w^u$. Skilled labor is used for two tasks: the amount $l^s$ is used for the production of output and the remainder, $h^s$, is used for the production of intangibles. Skilled labor is paid $w^s$. The production of intangibles is governed by the following technology:

$$x^I = \theta h^s$$

where $\theta$ is a productivity parameter. The existence of this function is motivated by the need to capture the link between the rise in the wage paid to skilled labor and the increase in the price of intangible investment goods. This function will allow the ratio of the intangible to tangible price to vary over time instead of being set equal to one as done in Hall (2001). Skilled labor is made of university graduates and executives. How these workers produce intangibles will be explained in the calibration section. The production of output proceeds according to $F(k_{t-1}, l^u_t, l^s_t)$ where $F(.)$ is assumed to be homogeneous of degree one. The price of output is set to be the numeraire.

The model departs from the baseline q-theory model by defining a composite investment good which is accumulated over time into a capital stock. The firm combines intangible goods $x^I$ with tangible investment goods $x^T$ to produce a final investment good according to

$$x = (x^T)^\gamma (x^I)^{1-\gamma}.$$  

This composite investment good captures the embodiment of intangibles in tangible investment goods. The exponent is allowed to vary over time in a deterministic fashion and is calibrated to capture the evidence on the increasing importance of intangibles relative to tangible investment. Tangible investment goods are bought from the market at a price $p^T$ which is taken as given by the firm. The price of intangible capital is denoted $p^I$ and will be
given by

\[ p^I = \frac{w^s}{\theta}. \]

The construction of this unit cost will be discussed in the calibration section, once the wage of skilled workers is specified as a function of the wage of university graduates and executives. As will be noted, the firm will take this price as given as well.

Note that the technology of production of the final investment good can be considered as an aggregator of two intermediate investment goods since it is a share-weighted function that is apparent to an investment index. The weights represent the share of each intermediate investment good in the overall investment expenditure. In fact, it is a Divisia index approach to combining two investment goods. Following this logic, \( x \) can be viewed as an index of aggregate investment.

The aggregate investment good accumulates according to

\[ k_t = (1 - \delta)k_{t-1} + x_t \quad (1) \]

where \( \delta \) is the depreciation rate. The adjustment of the capital stock is subject to output losses modeled as a cost function assumed to be quadratic and homogeneous of degree one that is denoted by \( C(x_t, k_{t-1}) \).

At each period, firm’s profits is given by

\[ v_t = F(k_{t-1}, l_t^n, l_t^s) - w_t^u l_t^n - w_t^s l_t^s - w_t^s h_t^s - p_t x_t^T - C(x_t, k_{t-1}). \]

The firm’s problem is to choose the optimal level of labor and investment in order to maximize the net present value of future profits subject to: 1) the technology of production of intangibles and of the final investment good, 2) the capital accumulation equation, 3) the starting level of capital and 4) the transversality condition:

\[
\max_{\{l_t^n, l_t^s, h_t^s, x_t^T, x_t\}} \hat{\nu}_s = \sum_{t=s}^{\infty} \left( \frac{1}{1 + r} \right)^{t-s} v_t \\
\text{s.t.} \quad x_t^I = \theta_t h_t^s \\
\quad \quad x_t = (x_t^T)^\gamma (x_t^I)^{1-\gamma} \\
\quad \quad k_t = (1 - \delta)k_{t-1} + x_t
\]
The value function $\hat{v}_s$ is the net present value at time $s$ of future payout to securities’ holders: after the firm pays inputs their due, the leftover income is paid to owners. Their ownership materializes through the possession of titles in the form of securities. Hence, $\hat{v}_s$ is also the value of the firm at time $s$.

The model can be shown to be equivalent to a standard q-theory optimization problem through a two stage optimization procedure. The only difference will lie in the interpretation of the price of investment goods. The first stage is a static problem which consists in choosing $x^T$ and $x^I$ to minimize the expenditure on the production of $x$ within each period. The second stage recasts the above dynamic problem accordingly such that it is solved at the start.

The static problem can be written as

$$\min_{x^T, x^I} p^T x^T + p^I x^I$$

subject to

$$(x^T)^\gamma (x^I)^{1-\gamma} \leq x.$$

Replacing the optimal solutions $x^{T*}$ and $x^{I*}$ into the objective function leads to the minimum cost function:

$$p^T x^{T*} + p^I x^{I*} = \left(\frac{p^T}{\gamma}\right)^\gamma \left(\frac{p^I}{1-\gamma}\right)^{1-\gamma} x \quad (2)$$

$$= p^x x \quad (3)$$

where $p^x$ reflects the unit cost of an investment good or the price index for aggregate investment. The new dynamic problem of the firm can be written as:

$$\max_{\{l^*_t, k^*_t, x_t\}} \hat{v}_s = \sum_{t=s}^{\infty} \left(\frac{1}{1+r}\right)^{t-s} v_t$$

subject to

$$k_t = (1-\delta)k_{t-1} + x_t$$

$$\lim_{T \to \infty} \left(\frac{1}{1+r}\right)^T \hat{v}_{s+T} = 0$$
where at each period, firm’s profits is given by
\[ v_t = F(k^x_{t-1}, l^w_t, l^s_t) - w^u_t l^w_t - w^s_t l^s_t - p^x_t x_t - C(x_t, k_{t-1}). \]

The solution to this standard problem is detailed in Appendix A. The usual first order condition on the equality of the lifetime return to increasing capital by one unit with its marginal cost is given by
\[ \lambda_t = p^x_t + C_x(x_t, k_{t-1}) \]
where \( \lambda_t \) is the shadow price of a unit of installed capital. The right hand side is the marginal cost given by the summation of the acquisition price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. In order to obtain sharper results with respect to the investment decision of the firm, the adjustment cost function is specified as quadratic and homogeneous of degree one, as is often done in the literature:
\[ C(x_t, k_{t-1}) = \alpha \left( \frac{x_t}{k_{t-1}} \right)^2 k_{t-1}. \]
Substituting this function into the first order condition results in the following equation:
\[ \frac{x_t}{k_{t-1}} = \frac{1}{\alpha} (\lambda_t - p^x_t). \] (5)
This is known as the investment equation since it relates the behavior of the investment rate to the difference between the value of capital in place \( \lambda_t \) and its acquisition cost \( p^x_t \). Investment is positive when the lifetime return to increasing capital by one unit exceeds its marginal cost and vice versa. To get around the fact that \( \lambda_t \) is by definition unobservable, the finding of Hayashi (1982) that
\[ v_t = \lambda_t k_t \]
is used to obtain the following expression:
\[ \frac{x_t}{k_{t-1}} = \frac{1}{\alpha} (v_t - p^x_t). \] (7)
Finally, in order to quantitatively implement the model, this relationship is combined with the expression for the investment term \( x_t \) to obtain the following quadratic equation:
\[ \alpha k^2_t + (p^x_t - \alpha (1 - \delta) k_{t-1}) k_t - v_t k_{t-1} = 0 \] (8)
Hall (2001) shows that a unique solution exists for a general convex cost function with constant returns to scale. This equilibrium is stable and is therefore not sensitive to initial conditions in the long-run. \( k_t \) is the endogenous variable to be solved for and generated at each point in time. The positive root expresses the law of motion of the capital stock:

\[
k_t = \frac{-[p_t^x - \alpha(1 - \delta)]k_{t-1} + \sqrt{([p_t^x - \alpha(1 - \delta)]k_{t-1})^2 + 4\alpha v_t k_{t-1}}}{2\alpha}.
\]

All variables are observable and the pair \((v_t, p_t^x)\) is a sufficient statistic to generate the stock of capital in the economy. Obviously, this pair of variables is taken as given by the firm.

Once a series for the capital stock is obtained, the contribution of each type of investment to overall capital stock can be recovered in the following way. The capital accumulation equation \( k_t = (1 - \delta)k_{t-1} + x_t \) is substituted forward to obtain:

\[
k_{t+T} = (1 - \delta)^{T+1}k_{t-1} + \sum_{i=0}^{T} (1 - \delta)^i \{x_{t+T-i}\}.
\]

Since the technology of production of the final investment good is homogeneous of degree one, the Euler theorem applied to \( x_t \) leads to:

\[
x_t = \frac{\partial x_t}{\partial x_t^T} x_t^T + \frac{\partial x_t}{\partial x_t^I} x_t^I = \left(\frac{p_t^T}{p_t^x}\right) x_t^T + \left(\frac{p_t^I}{p_t^x}\right) x_t^I.
\]

It is now possible to link the stock of capital to investment over time in each type of investment good by substituting this last relationship in the capital accumulation equation:

\[
k_{t+T} = (1 - \delta)^{T+1}k_{t-1} + \sum_{i=0}^{T} (1 - \delta)^i \left\{\left(\frac{p_{t+T-i}}{p_{t+T-i}^x}\right) x_{t+T-i}^T + \left(\frac{p_{t+T-i}}{p_{t+T-i}^x}\right) x_{t+T-i}^I\right\}.
\]

This expression will be used to disentangle the contribution of each type of investment to overall capital stock. Each type of investment is weighted by its relative price allowing the capital stock to be expressed in efficiency terms. This is a consequence of the aggregation formulation that was assumed for the final investment good.
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustment-cost</td>
<td>$\alpha$</td>
<td>8</td>
<td>Shapiro (1986)</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.026</td>
<td>Hall (2001)</td>
</tr>
<tr>
<td>Initial capital stock</td>
<td>$k_{t-1}$</td>
<td>$v_{t-1}$</td>
<td>Assuming $g_{t-1} = 1$ at $s = t - 1$</td>
</tr>
</tbody>
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4 Calibration

4.1 Constant Parameters

The parameter values and paths of some exogenous parameters in the law of motion of capital need to be specified.

The adjustment-cost parameter $\alpha$ represents the time it takes for the capital stock to double (halve) when $\lambda$ doubles (halves). To see this, note that if $\lambda$ doubles permanently, say from one to two, it will initially cause the investment-capital ratio to increase by $\frac{1}{\alpha}$. For the investment-capital ratio to double, the increase in $\frac{1}{\alpha}$ must be repeated for $\alpha$ periods. Hall (2001) cites the work of Shapiro (1986) to justify the choice of a doubling time parameter of 8 quarters. The depreciation rate of 2.6% per quarter is used by national income accounts for physical capital. Finally, to start the iteration on the law of motion of capital, the value of the initial capital stock $k_{t-1}$ needs to be specified. We will assume that at the pre-initial quarter, the value of the firm reflects its quantity of installed capital i.e. $k_{t-1} = v_{t-1}$. This is similar to assuming that $\lambda_{t-1} = 1$. Since the recursion was shown to be insensitive to initial conditions, this assumption will not affect the behavior of the system in the long-run.

Table 1 summarizes the parameter values used and the rationale for the choice of each value.

4.2 Varying Parameters

The market value of net financial claims (financial liabilities minus financial assets) is used as the measure of $v_t$ since the value of ownership claims are a reflection of the installed capital inside the firm. Indeed, $v_t$ was defined as the present value of payouts to securities’ holders. Assuming that investors are rational, it follows that the present value of payouts $v_t$ will equal the value of securities on the market. Since for all $t$, $v_t = \lambda_t k_t$, then the value of
securities equals the value of the installed capital stock.

Notice that \( v_t \) includes all financial claims towards firms, net of financial assets. These claims are made of equity, bonds and all other other liabilities (loans and mortgages, short-term paper, trade payables, life insurance and pensions). The definition of \( v_t \) represents a departure from most of the q-theory literature’s interpretation of \( v_t \). Traditionally, \( v_t \) covers only equity values or equity plus bonds. This departure is mainly due to the new types of data that are available for use.

Most of the data to measure \( v_t \) is taken from the national balance sheet account at market value from 1950Q1 to 2005Q4. Equity is reported at market value and all the other liabilities are at book value. These were converted by Hall (2001) into market value. The analysis focuses on the non-farm, non-financial corporate sector. This sector was chosen because it best fits the perfectly competitive framework of this paper. The removal of the farming sector aims to control for the presence of land in the overall capital stock, a capital input in fixed supply, which therefore earns rents. The choice of the corporate sector ensures that securities are continually priced to accurately reflect new information regarding the value of the capital stock. This would not be true for the installed capital of unincorporated businesses. Another reason to focus on this sector is dictated by the fact that the farming sector, the non-corporate sector and the financial sector suffer from data quality problems. The use of the non-farm, non-financial corporate sector is not restrictive given that this sector owns around 90% of the non-residential fixed capital stock in the economy.

The paper takes the view that intangibles are being produced by a class of skilled workers. This class is made of two broad categories of workers: on the one hand, scientists and middle managers that we categorize as university graduates and denote \( (h_t^{Univ}) \) who create raw intangibles and on the other hand, executives \( (h_t^{Exec}) \) who create organizational designs and structures. Raw intangibles and organizational designs are combined to produce intangibles following

\[
x^I_t = G_t(h^*_t) = \theta_t(h_t^{Exec})^{\phi_t}(h_t^{Univ})^{1-\phi_t}.
\]

(13)

Assuming perfect competition, the expression of the unit cost of an intangible capital good is then given by

\[
p^I = \frac{1}{\theta_t} \left( \frac{w_t^{Exec}}{\phi_t} \right)^{\phi_t} \left( \frac{w_t^{Univ}}{1-\phi_t} \right)^{1-\phi_t}.
\]

(14)
where $\theta$ is output per worker and $\phi$ is allowed to vary to reflect the variable weight that characterized executives compensation over time in intangible production’s overall wage bill:

$$
\phi_t = \frac{w_t^{Exec} h_t^{Exec}}{w_t^{Univ} h_t^{Univ} + w_t^{Exec} h_t^{Exec}}.
$$

(15)

This specification highlights the relationship between the labor market for high-skilled workers and the cost of intangibles. Since national income accounts do not collect information on the investment of firms in intangibles, and because the market for intangibles is extremely thin\(^7\), little is known about their aggregate price. The view taken in this paper is that the major manufacturers of intangible goods, as listed in Corrado et al. (2005), are university graduates and executives. Due to the rapid increase in demand of intangibles as documented by Corrado et al. (2006), this category of workers experienced a widely documented increase in their wage premium starting in the early 80s. (See Katz and Autor (1999) and Lemieux (2007) for an up-to-date review of this literature). Figure 3 shows the evolution of the real wage of university graduates from 1970 to today. The upward trend observed up to 2000 and the subsequent plateau is still preserved if this wage is expressed as a ratio of non-university graduates. Another category of workers which experienced an impressive rise in their wages

\(^7\)Some R&D spending leads to the creation of a patent which will carry a price if commercialized. However, the market for patents is extremely thin: very few patents change hands. For example, Serrano (2006) documents that only about 20% of all U.S. patents issued to small innovators (i.e., firms that were issued no more than five patents in a given year) are traded once or more.
Figure 4: Real Mean Compensation of Executives (in thousands). Source: Frydman and Saks (2007)

is executives. Frydman and Saks (2007) report data from 1935 on CEO compensation. Figure 4 reproduces their findings. There is an upward trend starting in the early 80s. The rise and fall in the stock market around 2000 seems to have had an important effect on the compensation trend.

This documented evolution in the labor market for high-skilled workers is important in valuing the competitive price of produced intangible goods and thereby accurately constructing an index for the price of aggregate investment. The initial value of $p^I$ is obtained by assuming that the firm is at steady-state at $t = 0$:

$$\frac{x_0}{k_{-1}} = \frac{1}{\alpha} \left( \frac{v_0}{p_0^x k_0} - 1 \right) \Leftrightarrow p_0^x = \frac{1}{\alpha \gamma}.$$  

Using $p_0^x = \left( \frac{p^I_0}{\gamma} \right)^{\gamma} \left( \frac{p^I}{1-\gamma} \right)^{1-\gamma}$ we can back out the value of $p^I_0$. $p^I_0$ is then made to grow at the rate of change of $p^I_t$.

Figure 5 depicts the implied behavior of $\phi_t$ overtime. In particular, it shows a downward trend from 1950 to early 70s reflecting the falling share of executive compensation in the overall wage bill. However, starting in the early 80s, the overall wage payments to this category of workers increases faster than the payments to university graduates up to 2000, fueled by the rise in the value of stock options held by executives. With the fall in the
Figure 5: Share of Executive Compensation $\phi_t$. Source: BLS, Frydman and Saks (2007) and author’s calculations.

value of securities, the wage bill to executives fell faster than that of university graduates explaining the fall observed after 2000. Note that the share reached in 2005 is similar to the starting level of 1950.

Using the wage of executives and college graduates together with the shares of these inputs allows the construction of the intangible unit cost. The behavior of this price is depicted in Figure 6 in log scale. The price of intangibles $p^I$ is rising at a very low constant rate from 1950 until mid-80s. Afterwards, its growth rate accelerates driven by the rise in the wage of CEOs and in their increasing share in the overall wage bill up to 2000. The growth rate of the intangible price falls afterwards.

It is possible to construct an aggregate investment price index that combines the price of intangible and tangible investment goods once the share of tangible expenditure in overall investment expenditures given by

$$\gamma_t = \frac{\bar{p}^T x^T}{\bar{p}^T x^T + \bar{p}^I x^I}$$

(16)

is calibrated. As mentioned before, Corrado et al. (2006) report the spending of U.S. firms on an identified list of intangible inputs. The National Income and Product Accounts (NIPA) recorded 20% of the reported expenditures in intangibles by Corrado et al. (2006). $p^T x^T$ is the private fixed investment as recorded by NIPA from which the included recorded intan-
gibles are subtracted. These recorded intangibles consisted of software, mineral exploration, and architectural and design services. The time series behavior of this ratio captures the biased technological change which resulted in the use of relatively more $x^I$. Figure 7 shows the behavior of $\gamma$ overtime.

As mentioned in the introduction, there is a striking upward trend beginning as early as the 1950s. After 2000, firms’ investment in intangibles began to surpass the investment in tangible investment goods. For the purpose of this paper, this is important in constructing an accurate measure of aggregate investment. Indeed, because the share of expenditure of firms in intangibles has been increasing overtime, the reported aggregate investment by national income accounts, which consists mainly of tangible investment, is only a partial reflection of the investment activity of firms.

It is now possible to use the behavior of the share series that was just constructed in

$$P^x_t = \left( \frac{p^T_t}{\gamma_t} \right)^{\gamma_t} \left( \frac{p^I_t}{1 - \gamma_t} \right)^{1 - \gamma_t}$$

(17)

to derive the series on the aggregate investment price index $P^x$. This is shown in Figure 8 together with the price series of tangibles $p^T$. Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose
behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts has a downward trend beginning in the mid-1950s, the investment deflator from the model has a downward trend until the mid-1980s and then rises until 2000 at which point it peaks and then falls\(^8\). In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for. In particular, note that the price of intangibles behaves opposite than that of the price of physical capital. This drives the unique shape of the aggregate price of investment since it is a shared weighted index.

5 Quantitative Findings and Results

5.1 Behavior of the Stock of Capital

Figure 9 shows how the value of the firm \(v_t\) is contrasted to the generated capital stock \(k_t\) and to its acquisition cost \(p^x_t k_t\).

The first notable feature is that the implied stock of capital smoothly increases until the peak of 2000 and then falls and flattens out. The second notable feature is the contrast

\(^8\) The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood \textit{et al.} (1997) and Krusell (1998).
Figure 8: Real Price of Investment Goods \( p^x = \left( \frac{p^x}{\gamma} \right)^\gamma \left( \frac{p^I}{1-\gamma} \right)^{1-\gamma} \). Source: NIPA and author’s calculations.

Figure 9: The Aggregate Value of Firms and the Inferred Capital Stock.
between the series of the acquisition cost of capital with the one shown in Figure 1 which uses the national accounts investment deflator. The smoothness depicted from official statistics in Figure 1 is at odds with what Figure 9 depicts. The rise in the acquisition cost of capital as a result of the larger share of intangibles and their rising cost is at the source of this discrepancy.

5.2 Implications for Tobin’s q Series

Tobin’s $q$ is the ratio of the value of an additional unit of capital \textit{in place} to the price of acquiring new capital:

$$
q_t = \frac{\lambda_t}{p^x_t}
$$

(18)

Since $v_t = \lambda_t k_t$, then

$$
q_t = \frac{v_t}{p^x_t k_t}
$$

(19)

The result from the extended $q$-theory shows that the behavior of Tobin’s $q$ is almost always positive. Figure 10 depicts the fact that the market value of the capital stock predominantly remains above and close to its acquisition cost; this reflects a Tobin’s $q$ that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin’s $q$ when capital is exclusively made of tangible investment. This comes as no surprise since the steady-state value of $q$ is $p^x_t + \delta \alpha$ which is around 1.2. Hall (2001) delivers the same result for the same reason although his $q$ is smaller on average because of the non-variation of $p^x_t$ which is set to one. Laitner and Stolyarov (2003) also deliver a theoretical $q$ that is positive due to how they specify the production of intangibles. Their model is not one of adjustment costs but nevertheless, their Tobin’s $q$ is expressed such that it is always above one.

5.3 Contribution of Intangibles and Tangibles to Capital Stock

Figure 11 shows the decomposition of the contribution of the series of intangibles and tangible capital. Notice that the contribution intangible investment goods has been substantial in the 60s and the 90s. This Figure shows also that the rise in overall capital stock from the mid-80s until the late 90s was mainly driven by an increase in intangible investment, underscoring the importance of intangibles as a source of value for the firm and as a key component of any
investment theory. In particular, intangibles contributed the same amount to overall capital stock by 2000 as did tangible investment goods.

6 Comparison of Findings with Existing Literature

In this section, I will compare the model developed in this paper with the models used both by Hall (2001) and by McGrattan and Prescott (2007a). The comparison will abstract from the presence of adjustment costs, taxes and difference in depreciation rates and will focus on the expression of the firm’s value and the implications of assumptions made about the measurement of the size of the capital stock. The results of each paper will also be contrasted with the findings described above.

Recall that the forward substitution in the capital accumulation equation led to the general expression given by equation 12, reproduced here for convenience:

$$k_{t+N} = (1 - \delta)^{N+1}k_{t-1} + \sum_{i=0}^{N} [(1 - \delta)^i x_{t+N-i}].$$

The first term tends to zero as $N$ becomes large. The expression can be re-written as

$$k_{t+N} = \sum_{i=0}^{N} (1 - \delta)^i x_{t+N-i}. \quad (20)$$

This expression will be shown to differ in important ways across approaches.
6.1 Reframing Hall (2001)

As mentioned in the introduction, the model developed in this paper differs from Hall (2001) by relaxing the assumption that $p^x = p^T$ and by allowing $x^T$ and $x^I$ to have some degree of substitutability. In the case of Hall (2001), the value of the firm is given by

$$V_t = p_T^T k_t = p_T^T (k_T^T + k_I^I)$$  \hspace{1cm} (21)

where $p_T$ is the (physical) investment price deflator from national accounts. Hall assumes that there is one price of investment goods i.e., $p^T = p^I$ and implicitly assumes that there are two equations for the capital stock at each point in time, one for each type of investment good:

$$\begin{align*}
k_{t+N}^T &= \sum_{i=0}^{N} (1 - \delta)^i x_{t+N-i}^T \\
k_{t+N}^I &= \sum_{i=0}^{N} (1 - \delta)^i x_{t+N-i}^I
\end{align*}$$  \hspace{1cm} (22)

6.2 Reframing McGrattan and Prescott (2007a)

In McGrattan and Prescott (2007a), their model implies an expression for the value of the firm that can be written as:

$$V_t = p_T^T k_t^T + p_I^I k_t^I = k_T^T + p_I^I k_I^I$$  \hspace{1cm} (23)
where $p_T^t$ is the price of tangible investment goods (considered to be equal to the numeraire, assumed to be the consumption good) and $p_I^t$ is the price of intangible capital goods. The price of intangibles is guessed as part of the solution to their general equilibrium model. Unfortunately, the authors do not report the series for $p_I^t$. The model of McGrattan and Prescott features two capital accumulation equations similar to expression 22, as done by Hall as well.

6.3 Models’ Comparison

In this paper, the value of the firm without adjustment costs can be written as

$$ V_t = p_t^x k_t = \left( \frac{p_T^t}{\gamma_t} \right)^{\gamma_t} \left( \frac{p_I^t}{1 - \gamma_t} \right)^{1-\gamma_t} k_t. $$

Note that the two investment goods in this paper are not accumulated because they are intermediate goods. However, in Hall (2001) and in McGrattan and Prescott (2007a), the two investment goods are final so they are accumulated over time. Moreover, the non-embodiment in Hall and in McGrattan and Prescott of investment goods results in two prices, one for each accumulated investment good while in this paper, there is only one price for the accumulated investment good. This one aggregate price deflator is attached to a single overall stock of capital, as it is commonly modeled when using an aggregate production function.

The comparison of the expressions for capital stock in each model will also reveal another consequence of the embodiment hypothesis. To see this, note that the embodiment of intangibles is reflected when equation 20 is expanded after using the Euler theorem on $x_{t+N-i}$:

$$ k_{t+N} = \sum_{i=0}^{N} (1 - \delta)^i \left\{ \left( \frac{p_T^{t+N-i}}{p_T^{t+N-i}} \right) x_{t+N-i}^{T} + \left( \frac{p_I^{t+N-i}}{p_I^{t+N-i}} \right) x_{t+N-i}^{I} \right\} $$

$$ = \sum_{i=0}^{N} (1 - \delta)^i \left( \frac{p_T^{t+N-i}}{p_T^{t+N-i}} \right) x_{t+N-i}^{T} + \sum_{i=0}^{N} (1 - \delta)^i \left( \frac{p_I^{t+N-i}}{p_I^{t+N-i}} \right) x_{t+N-i}^{I}. $$

Both parts of the right-hand-side of this last expression are similar to the corresponding equations for Hall (2001) and for McGrattan and Prescott (2007a) when we allow $p^T = p^I$. In this case,

$$ k_{t+N} = k_{t+N}^T + k_{t+N}^I. $$
Such capital stock exists in Hall (2001) by construction but not in McGrattan and Prescott (2007a) because this is not that capital stock which enters into the production of intangibles or in the production of final output. This is a major theoretical difference.

6.4 Comparison of Findings

Hall’s (2001) implied capital stock series has a very pronounced bell-shape around 2000. Given that the acquisition cost of capital goods is identified to the tangible deflator which was still falling throughout, his model’s implication of a fall in the value of securities is for the firm to disinvest. That’s not the case in the extended model: the aggregate acquisition cost of capital becomes heavily skewed towards intangibles whose price begins to rise starting in the 80s. It then falls after 2000. The implications is that the firm in this paper has less incentive to accumulate capital before 2000 and to disinvest after 2000 once the price of aggregate investment falls. This explains why the extended model features a much less pronounced bell-shape around 2000.

The assumption of the equality between the prices of intangibles and tangibles leads to Hall’s finding that the quantity of intangible capital is negative from the mid-70s until the mid-80s. The model in this paper does not feature such an anomaly. To see why, observe that the constructed price series of aggregate investment in Figure 8 falls faster than the tangible price series up until the mid-80s. Firm’s incentive to invest in the extended model is therefore bigger than in Hall’s model. As a result, the accumulation of capital is higher during this period than in Hall, which makes and the implied contribution of intangible investment is higher.

Comparing the extended model’s results with the findings of McGrattan and Prescott (2007a) is not as straightforward because they use a general equilibrium framework and because they emphasize different series. In particular, the price of intangibles is not reported in their paper. In their technical appendix paper (McGrattan and Prescott (2007b)) the authors only mention that they guessed this price series. Furthermore, McGrattan and Prescott do not report a series for intangible investment alone, which makes it hard to compare the size of intangible investment directly. However, the authors report two series that include, in one way or another, intangible investment: 1) intangible investment as a
share of total GDP (their Figure 6) and 2) a graph containing the series of overall investment and tangible investment (their Figure 12). It is not possible to use the first series to make quantitative comparisons regarding the amount of intangible investment because McGrattan and Prescott make many corrections to GDP. However, this series can be used to compare the qualitative findings. In particular, McGrattan and Prescott find that intangibles are falling in the early 90s while in this paper intangible investment was shown to rise during that period. The behavior of intangible investment around 2000 is similar. The second series is used to approximate the difference between the overall investment and the tangible investment which is the intangible investment found in McGrattan and Prescott. Following this approach, intangible investment seems quite small between 1990 and 1994. It then rises until 1999; This rise is about 20%. Afterwards, intangible investment falls to a very low level. The drop is about 40% between 2000 and 2003. On the other hand, the extended q-theory model results in a rise of intangible investment by 150% between 1990 and 1992. Then it falls by 150% between 1992 and 1994. After 1994, intangible investment increases until 1999 by 300% and falls afterwards by 350%. The bottom line of these comparisons is that the extended model reports more intangible investment growth before 2000 and more decline after 2000 than that in McGrattan and Prescott (2007a).

7 Conclusion

This paper extended the q-theory of investment to account for intangible investment and used the model to measure the contribution of embodied intangible goods to overall capital stock in the U.S. The paper also explored the quantitative implications of the model on the behavior of the capital stock and Tobin’s q.

The price trend of aggregate investment was shown to contrast markedly with the aggregate investment deflator series reported by national accounts because this deflator series does not account for the price effect of intangibles. In addition, the extended model successfully generated a smoothly-behaving series of capital stock with a market value that predominantly remains above its book value; this reflects a Tobin’s q that is mostly above one, a feature which is not observed in empirical measures of Tobin’s q when capital is exclusively made of tangible investment. Furthermore, the model shows that the rise in
overall capital stock from the mid-80s until the late 90s was mainly driven by an increase in intangible investment. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the price movement of intangibles beyond simply their rising share in overall investment. These findings confirm the changing nature of the stock of capital in the economy and the quantitative importance of intangibles, underscoring the importance of intangibles as a source of value for the firm and as a key component of any investment theory.

Three main avenues are proposed for future work. First, the price of aggregate investment reflected two secular stylized facts: 1) the larger role for intangibles in production and 2) the rise in their cost of production as illustrated by the growth in compensation of skilled labor and executives. The link between the labor market for high-skilled workers and capital markets was shown to be key in obtaining these results. This feature might provide a better understanding of the mixed performance of the econometric estimation of the investment equation. These regressions tend to produce low $R^2$’s and serially correlated residuals. Moreover, additional regressors, such as output and cash flow, also appear to be important factors in the investment decision, as they typically have statistically significant coefficients (Chirinko (1993)). These problems may be due to the omission of intangibles when valuing the true cost of new investment goods. In future work, I plan to use the data series on the acquisition cost of capital corrected for the inclusion of intangibles, as constructed above, to investigate whether the econometric estimation of the investment equation did indeed suffer from specification problems.

A second future extension is to explore the insight that intangibles are produced mainly by skilled workers, in order to address the puzzle found in Hall (2003). In that paper, Hall finds that the payment to intangibles is almost nil. He reaches this conclusion after comparing the income stream accrued from physical capital as predicted by the baseline user-cost of capital theory, with the income stream observed from earnings of firms. Hall finds that the present value of the future flow of income generated by intangibles is close to zero, suggesting that their size is unimportant. I suspect that the treatment of intangibles as expenditures (as opposed to capital goods) caused reported earnings to be undervalued. This omission could deliver the finding that once physical capital is paid for its services,
there is nothing left to reward intangible capital. I plan to redo Hall’s (2003) exercise using earnings that are corrected for this mismeasurement and investigate whether Hall’s findings hold for this correction.

Finally, part of the value of the firm includes a portion that is made up of rents. If this portion is large then the baseline model could be confusing some of the capital accumulation for rent accumulation with the consequence of underestimating the shadow price of capital. An analogous implication would be the overestimation of the quantity of capital. I plan to address this issue by extending the framework of this paper to accommodate for the existence of rents and measure their economic importance.
References


Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, “Long-Run Implications of


Appendix

A Derivation of First-Order Conditions

The firm’s problem is

\[
\max \{ l^u_t, l^s_t, x_t \} \quad \text{s.t.}
\]

\[
v_t = F(k^x_{t-1}, l^u_t, l^s_t) - w^u_t l^u_t - w^s_t l^s_t - p^x_t x_t - C(x_t, k_{t-1})
\]

\[
\lim_{T \to \infty} \left( \frac{1}{1 + r} \right)^T \hat{v}_{s+T} = 0.
\]

The Hamiltonian \( \mathcal{H} \) at time \( s \) and the first order conditions are given by

\[
\mathcal{H}_s = \sum_{t=s}^{\infty} \left( \frac{1}{1 + r} \right)^{t-s} \{ F(k^x_{t-1}, l^u_t, l^s_t) - w^u_t l^u_t - w^s_t l^s_t - p^x_t x_t - C(x_t, k_{t-1}) - \lambda_t [k_t - (1 - \delta) k_{t-1} - x_t] \}
\]

\[
\frac{\partial \mathcal{H}_t}{\partial x_t} : \quad \lambda_t = p^x_t + C_x(x_t, k_{t-1})
\]

\[
\frac{\partial \mathcal{H}_t}{\partial l^u_t} : \quad w^u_t = F_{l^u}(k^x_{t-1}, l^u_t, l^s_t)
\]

\[
\frac{\partial \mathcal{H}_t}{\partial l^s_t} : \quad w^s_t = F_{l^s}(k^x_{t-1}, l^u_t, l^s_t)
\]

\[
\frac{\partial \mathcal{H}_t}{\partial k_t} : \quad \lambda_t (1 + r) = F_k(k^x_{t-1}, l^u_t, l^s_t) - C_k(x_{t+1}, k_t) + (1 - \delta) \lambda_{t+1}
\]

\[
\frac{\partial \mathcal{H}_t}{\partial \lambda_t} : \quad k_t = (1 - \delta) k_{t-1} + x_t
\]

where \( \lambda \) is the costate variable or the shadow price of an additional unit of capital. The first equation illustrates the equality of the lifetime return to increasing capital by one unit with its marginal cost given by the price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to
be chosen by the firm. The second and third equation state the usual equilibrium condition for the labor market whereby the real wage is equal to the marginal product of labor. The next equation shows the dynamic equilibrium equation of $\lambda$ with its continuation value. The last equation recasts the investment technology constraint.

## B Allowing for Irreversibility in Investment

The generated stock of capital is not sensitive to the existence of irreversibility in investment. It is possible to assume that the cost function is piece-wise quadratic:

$$C(x_t, k_{t-1}) = \begin{cases} \frac{\alpha^+}{2} \left( \frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t > 0 \\ \frac{\alpha^-}{2} \left( \frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t < 0 \end{cases}$$

where the adjustment-cost parameter $\alpha^+ (\alpha^-)$ has the same interpretation as in the model: it represents the time it takes for the capital stock to double (halve) when $\lambda$ doubles (halves). By allowing the downward adjustment-cost parameter to be higher than the upward adjustment-cost parameter, this asymmetry in the investment decision will reflect irreversibility of investment. Setting $\alpha^+ = 8$ as in the main text and allowing $\alpha^-$ to be arbitrarily set at up to ten times higher than the upward adjustment-cost parameter, the generated capital stock will not be affected. The result behind this finding comes from the fact that gross investment is always above net investment in the data.
C  Data Sources and Definitions

The deflator used is the quarterly CPI and the base year adopted is 1996. This series is obtained from the Bureau of Labor Statistics which reports the CPI on a monthly basis for all urban consumers from 1913 until today (Series ID: CUUR0000SA0). I take the average of three consecutive months to obtain the quarterly equivalent.

The data on output per worker are published by the Bureau of Labor Statistics under the heading “major sector productivity and costs index”. Because of data availability, I use the output per person of the business sector from 1950 until 1957 (Series ID: PRS84006163) and then the output per person of the non-financial corporate sector from 1957 until 1975 (Series ID: PRS88003163).

The data on wages of workers by educational attainment is collected annually by the Bureau of Labor Statistics in the Current Population Survey and reported every March for the whole economy. I use table A-3 entitled "Mean Earnings of Workers 18 Years and Over, by Educational Attainment, Race, Hispanic Origin, and Sex: 1975 to 2005" to obtain data on mean annual earnings and number of workers by educational attainment. There are 5 educational levels: not a high school graduate, high school graduate, some college/associate degree, bachelor’s degree and advanced degree. Earnings refer to the total income people receive for work performed as an employee during the income year. This includes wages, salary, armed forces pay, commissions, tips, piece-rate payments, and cash bonuses earned, before deductions are made for items such as taxes, bonds, pensions, and union dues. The wage of university workers is calculated as follows. I multiply the mean annual earnings of workers with a bachelor’s degree and advanced degree by their respective number of workers and divide the result by the total number of workers with a bachelor’s and advanced degree. I extend the data from 1975 until 1950 using the growth of output per worker. The implicit assumption is that the mean earnings of university graduates grew at the same rate as productivity per worker from 1950 to 1975. This assumption has a strong empirical support during this period (see Lemieux (2007)).

The wage bill of skilled labor from 1975 to 2005 is obtained by multiplying the mean earnings of university graduates by the number of university graduates. I extend the data back to 1950 using the economy wide compensation of employees. This series is provided by
the Bureau of Economic Analysis in their Table 1.10. entitled “gross domestic income by type of income”, line 2. This extension relies on the same assumption as the one outlined in the previous paragraph.

The data on executive compensation and its composition are taken from Frydman and Saks (2007). This series is a major improvement on previous studies which collected data on executive compensation for short samples, with different sample designs and employed different methodologies to value compensation and its components. (See for example Antle and Smith (1985), Hall and Liebman (1998) and Bebchuk and Grinstein (2005)). The work of Frydman and Saks (2007) is the first comprehensive panel dataset on executive compensation that spans the period 1936 to 2003. The sample follows the compensation of individual officers in the largest 50 publicly traded corporations ranked according to the value of sales in 1940, 1960 and 1990. This amounted to a total of 102 firms. Frydman and Saks discuss the representativeness of their sample in Appendix Section 3, and conclude that it is representative of the largest 300 publicly-traded corporations. They limit their analysis to the top three officers in order to maintain a consistent group of individuals over time, but the results are robust to including the 4th and 5th highest-paid executives.

The data on the number of chief executives is taken from the Occupational Employment Statistics (OES) Survey produced by the Bureau of Labor Statistics. The occupational title is “chief executive” with the occupational code 11-1011. This data is available from 1998 to 2005. From 1983 until 1997, the OES survey used a somewhat different classification system. The closest occupational definition for our purpose is the “management, business, and financial operations occupations” (Series ID: LNU02032202). I use the growth rate of this occupation to extend the data on the number of chief executives backward to 1983. Finally, I use the growth rate of the employment in the private sector to extend this data backward to 1950. This data are produced by the Bureau of Economic Analysis under Table 6.5 A-D with the heading “full-time equivalent employees by industry”.

The wage bill of executives is calculated by multiplying the average CEO compensation reported by Frydman and Saks (2007) by the number of chief executives described in the previous paragraph.

The price of tangible investment goods is the national income and product accounts
implicit deflator for fixed non-residential investment. This series is published by the Bureau of Economic Analysis under Table 7.1 “quantity and price indexes for GDP quarterly”, line 32.

The data on the real capital stock and the value of securities are taken from Hall (2001) and are extended to 2005. The data on intangible expenditures are taken from Corrado et al. (2006).