

# Expectations and Social Preferences in the Investment Game: A Partial Identification Approach <sup>\*</sup>

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## Abstract

We show how bounds around preferences parameters can be estimated under various levels of assumptions concerning the beliefs of senders in the investment game. We contrast these bounds with point estimates of the preference parameters obtained using data on the subjective beliefs of players. Our point estimates suggest that expected responses and social preferences both play a significant role in determining investment in the game. Moreover, these point estimates fall within our most reasonable bounds. This suggests that inferences based on stated belief data are reasonable.

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## 1 Introduction

A recent development in econometrics concerns the identification and estimation of econometric models which are partially identified (see Manski and Tamer (2002)). A model is partially identified if it maintains a weaker set of assumptions than are needed to point identify the parameters of interest. Maintaining weaker assumptions has the benefit of providing more robust inferences. The main drawback is

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<sup>\*</sup>An OX code with files implementing all the procedures discussed in this paper can be downloaded at <http://www.ecn.ulaval.ca/charles.bellemare/>. We thank Jim Cox and the Economic Science Laboratory in Tucson, Arizona, for financial and technical support, Urs Fischbacher for his support in programming the experiment, and Wafa Hakim for her research assistance in conducting the experiment.

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that such models can at best place bounds around the model parameters of interest. These bounds can be used to define the identification region of the model parameters given the data. This region contains all vectors of model parameters which are consistent with the data under weak assumptions concerning the key variables. The identification region can in turn be used to perform specification tests of the validity of maintaining stronger assumptions to point identify the model parameters. In particular, maintaining stronger but invalid assumptions concerning key variables may yield point estimates which fall outside the identification region. Applications of these methods have appeared in various areas in economics including health (Manski and Tamer (2002)) and industrial organization (Ciliberto and Tamer (2009)). To our knowledge, these methods have yet to be applied in behavioral economics.

In this paper we illustrate the usefulness of these methods by making inferences on preferences in a choice problem with uncertainty when researchers maintain minimal assumptions on the beliefs of players. More specifically, we specify a simple model of sender behavior in a binary investment game (see Berg, Dickhaut, and McCabe (1995)). We model preferences of senders as a function of their expected final payoffs (which proxies their trust in the responder), their concerns for social efficiency, and an unobserved random component. We focus on relating the size of the identification regions to the restrictiveness of the assumptions maintained on the beliefs of players. To do so, we compute identification regions around our model parameters under three different sets of assumptions about the beliefs of senders. The first and weakest set of assumptions consists of assuming that researchers have no information about beliefs of senders apart from the natural restrictions imposed by the game (eg. the amount returned must be below and above known boundaries). The second set of assumptions consists of assuming that all senders believe that the amount returned by the responder is no less when they invest than when they do not invest. This second set is more restrictive than the first. As a result, we expect the identification region under the second set to be contained in the identification region derived under the first set of assumptions. The third and most restrictive set of as-

assumptions we consider consists of assuming that senders have rational expectations. We show that the latter set of assumptions produce the smallest identification region of the three we consider.

In the last section of the paper we obtain point estimates of our model parameters using beliefs stated by senders in the experiment. Our point estimates suggest that expectations about responder behavior as well as social preferences are both significant determinants of investments. Moreover, we find that our point estimates fall within the first two identification regions. As a result, we are unable to reject the validity of using stated belief data to make inferences on preferences in this setting. In other words, elicited beliefs do not only reduce the identification region, but also appear to provide reasonable results.

These results relate to recent attempts to determine whether investments in this game reflect genuine trust that responders will return money to senders rather than underlying social preferences. Cox (2004) uses a design which exploits data from multiple treatments to investigate this issue. In particular, he rules out uncertainty in some treatments by preventing responders to make a choice. He shows how data from these multiple treatments can be used to separate trust from social preferences. Here, we exploit subjective belief data to separate trust (ie. expected responder behavior) from social preferences using a single treatment.

The rest of the paper is organized as follows. Section 2 presents the experimental design and the data. Section 3 the econometric model. Section 4 presents our results. Section 5 concludes.

## **2 Experimental design and procedures**

Our experimental design is a modified version of the two player investment game of Berg, Dickhaut and McCabe (1995). In our experiment, senders and responders

were both endowed with 6\$US.<sup>1</sup> Contrary to Berg, Dickhaut and McCabe (1995), we restricted the decision space of senders to two choices: investing all or none of the endowment. If a sender invested his endowment, that amount was doubled and added to the endowment of the responder. In turn, the responder had the opportunity to return any amount from his augmented endowment to the sender (ie. he could return up to 18\$).<sup>2</sup> If the sender did not invest his endowment, the responder could return any amount from his initial endowment (up to 6\$).

Responders made their decisions using the strategy method: they each had to decide how much to return when the sender invests his endowment, and how much to return when the sender does not invest his endowment. The decision which corresponded to the actual choice of the sender was chosen to be the effective action and determined the payoff of both participants. After making their decisions, senders were asked to state their subjective beliefs. Before stating their beliefs, they were further reminded of the decision tasks and given examples to clarify the belief elicitation procedure. Senders were not rewarded for the accuracy of their beliefs.

Proposers had to state their subjective beliefs in two scenarios. They were first asked to state their beliefs if they did not invest. In particular, they had to state how many out of 100 responders would return 0\$, and how many would return amounts in the following intervals  $\{(0, 1], (1, 2], (2, 3], (3, 4], (4, 5], (5, 6]\}$ .<sup>3</sup> By allowing senders to place a positive probability on getting back 0, we allow their subjective distribution functions to be censored from below. Additionally, senders were asked to state their beliefs about responder behavior if they invested their endowment. Senders were asked to state how many out of 100 responders would return 0\$, and how many would return amounts in the following intervals  $\{(0, 3], (3, 6], (6, 9], (9, 12], (12, 15],$

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<sup>1</sup>The complete content of the computer screens can be downloaded from <http://www.ecn.ulaval.ca/charles.bellemare/>.

<sup>2</sup>Expanding the choice set of senders is in principle possible, but this will require asking each participant to answer many more questions on their beliefs (see below).

<sup>3</sup>If the probability mass entered exceeded 100, senders were automatically instructed to go back and adjust their answers.

(15, 18]}.<sup>4</sup>

After all participants had made their decisions, senders and responders were randomly matched and payoffs were computed based on the decisions of the pair. Participants were then informed of the outcome of the experiment and their final payoffs. The experiment was conducted in May 2005 at the Economic Science Laboratory at the University of Arizona using the software zTree (Fischbacher (2007)). Participants were recruited via email and were mainly students in finance, business administration, economics, and engineering. Participants received a 5\$ show-up fee upon arrival at the laboratory. In total 122 participants interacted in 9 sessions of the experiment. We observed 38 pairs of players. An experimental session lasted on average 60 minutes, and, including their show up fee, participants earned on average 12.18\$ (9.92\$ for senders and 15.87\$ for responders).

24 of the 38 senders (63%) invested their endowment. To gain some insights on whether investors and non-investors trusted responders differently, we compare the distributions of beliefs of investors with the distributions of beliefs of non-investors ( $N = 14$ ). Figure 1 presents the average subjective belief distributions of investors (light bars,  $N = 24$ ) and non investors (dark bars,  $N = 14$ ). We find that both groups had similar beliefs about responder behavior if they consider not investing their endowment. In particular, both investors and non investors place a very high probability of getting nothing back from responders. Differences between both investors and non-investors emerge when we look at their beliefs in the event of investing their endowment. There, non-investors placed a 48.3% probability on getting nothing back from responders, substantially less than the 24.6% probability placed by investors as a

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<sup>4</sup>In order to detect whether senders stated beliefs to rationalize their decisions, we randomized approximately one third of all participants in our experiment to a group of “observers” who did not make any decisions but who answered the belief questions after having read the same instructions as all other participants. Observers received each 6\$ for their participation. We found no significant differences between the beliefs of senders and those of observers. See the extended working paper version of the paper for details (Bellemare, Bissonnette, and Kröger, 2007).

whole. Moreover, investors placed significantly more probability than non investors on getting back any positive amount. These results indicate that investors believed they would get higher returns when investing their endowment than non-investors.

To assess whether the beliefs of senders were rational, we computed for each sender their subjective expectations (when investing and not investing) in deviation from the observed average amount returned (0.208\$ when investing and 3.812\$ when not investing). Figure 2 presents the distributions of these differences. We find small discrepancies between expectations and observed responses when not investing, reflecting the fact that most senders correctly anticipated that the probability of getting close to nothing would be high when not investing. More substantial discrepancies emerge when considering amounts returned when investing. There, we find a substantial amount of senders have expectations below and above the observed amount returned.

### 3 A Simple Model of Choice

We assume that the utility of not investing for sender  $i$  is given by  $u_i^{keep} = \beta(w + r_i^{keep})$ , where  $r_i^{keep}$  denotes the amount the responder returns to sender  $i$  when  $i$  does not invest,  $w$  denotes the initial endowment of sender  $i$ , and  $\beta$  measures the marginal utility of income.<sup>5</sup> The amount returned when not investing  $r_i^{keep}$  can vary between 0 and the endowment  $w = 6\$$  of the responder.

When sender  $i$  invests, he foregoes his endowment  $w$  which is then doubled and transferred to the responder. As a result, a surplus of  $w$  is created when investing. We model the utility of investing as  $u_i^{invest} = \beta r_i^{invest} + \theta$ , where  $r_i^{invest}$  denotes the amount

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<sup>5</sup>To investigate whether the risk neutrality hypothesis is reasonable, we asked participants at the end of the experiment to play a sequence of lotteries similar to that proposed by Holt and Laury (2002). We found no significant relationship between measured risk preferences and investment behavior. Similar results have been found by Eckel and Wilson (2004) and Houser, Schunk, and Winter (2006).

returned by the responder when investing,<sup>6</sup> and  $\theta$  denotes the marginal utility of creating a surplus of  $w$  when investing. Recent studies suggest that concerns for social efficiency may be particularly important (see Engelmann and Strobel (2004)).<sup>7</sup> In terms of our model, this would imply that  $\theta > 0$ .

We next assume that senders make their decisions by comparing their subjective expected utilities of investing and not investing. The expected utilities of not investing and investing are given by

$$\mathbf{E}\left(u_i^{keep}\right) = \beta\left(w + \mathbf{E}\left(r_i^{keep}\right)\right) + \epsilon_i^{keep} \quad (1)$$

$$\mathbf{E}\left(u_i^{invest}\right) = \beta\mathbf{E}\left(r_i^{invest}\right) + \theta + \epsilon_i^{invest}, \quad (2)$$

where the expectations are computed with respect to the subjective distribution functions of sender  $i$ . To allow for the fact that some senders will make sub-optimal choices, we add standard normal error terms  $\epsilon_i^{invest}$  and  $\epsilon_i^{keep}$  to the true expected utilities  $\mathbf{E}(u_i^{invest})$  and  $\mathbf{E}(u_i^{keep})$ , and assume that first player  $i$  chooses the option  $j \in \{keep, invest\}$  that maximizes  $\mathbf{E}(u_i^j) + \epsilon_i^j$  rather than  $\mathbf{E}(u_i^j)$ .

## 4 Estimated identification regions

We first characterize the identification region of  $(\beta, \theta)$  which are consistent with the observed choice distribution of senders without imposing any information on beliefs. To estimate this region, we first consider the extreme case where all senders expect to receive with probability 1 the highest possible amount when investing ( $r^{invest} = 3w$ ) and the lowest possible amount when not investing ( $r^{keep} = 0$ ). This gives rise to the largest payoff difference between investing and not investing. In this case, the decision rule is to invest when  $\mathbf{E}(u_i^{invest}) > \mathbf{E}(u_i^{keep})$ , or equivalently

$$\beta(2w) + \theta + \epsilon_i > 0. \quad (3)$$

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<sup>6</sup>The amount returned  $r_i^{invest}$  by the responder can take a value between 0 and  $3w = 18\$$ .

<sup>7</sup>The preferences estimated are equivalent to preferences with linear altruism  $u_i = \gamma x_i + \alpha x_j$  with  $\gamma = \beta + \alpha$ ,  $\alpha = \theta/w$ , and where  $x_i$  and  $x_j$  respectively denote income of player  $i$  and  $j$ .

where  $\epsilon_i = \epsilon_i^{invest} - \epsilon_i^{keep}$ . A second extreme case occurs when all senders expect to receive with probability 1 the lowest amount possible when investing ( $r^{invest} = 0$ ), and the highest possible amount when they do not invest ( $r^{keep} = w$ ). This gives rise to the smallest payoff difference between investing and not investing. In this case, senders  $i$  will invest when

$$\beta(-2w) + \theta + \epsilon_i > 0. \quad (4)$$

Assuming that errors  $\epsilon_i$  are statistically independent of each other and follow a standard normal distribution, aggregating inequalities (3) and (4) across the population yields the following set of inequalities relating the population probability of investing to the model parameters

$$\Phi(\beta(-2w) + \theta) \leq \Pr(invest) \leq \Phi(\beta(2w) + \theta) \quad (5)$$

where  $\Phi(\cdot)$  denotes the standard normal cumulative distribution. The identification region for  $(\beta, \theta)$  contains all vectors of parameters that satisfy inequalities (5).

The shaded area in Figure 3 represents the identification region estimated by replacing  $\Pr(invest)$  with the proportion of investments observed in our sample. It is immediate from (5) that  $\theta$  is point-identified and equal to  $\Phi^{-1}(\Pr(invest))$  when expectations have no influence on the decision process ( $\beta = 0$ ). Otherwise, the observed proportion of investments is compatible with any combination of  $\beta > 0$  and  $\theta$  within the shaded area. We can easily see that the identification region of the social preference parameter  $\theta$  increases with  $\beta$ , the strength of the effect of expectations on investment behavior.

A smaller identification region can be derived by assuming somewhat realistically that  $\mathbf{E}(r_i^{invest}) \geq \mathbf{E}(r_i^{keep})$  for all senders. Under this assumption, inequality (3) remains unchanged as it does not violate the new restriction on beliefs. Inequality (4) on the other hand concerns the lowest possible payoff difference, a difference of 0 under the new restriction ( $\mathbf{E}(r_i^{invest}) = \mathbf{E}(r_i^{keep})$ ). In this case, senders  $i$  will invest when

$$\beta(-w) + \theta + \epsilon_i > 0. \quad (6)$$

Aggregating inequalities (3) and (6) across the population produces a new set of inequalities relating the population probability of investing and the model parameters

$$\Phi(\beta(-w) + \theta) \leq \Pr(\text{invest}) \leq \Phi(\beta(2w) + \theta). \quad (7)$$

The smaller identification region derived from (7) is given by the dark shaded area in the Figure (3). As expected, the new area is a strict subset of the area derived previously which places much tighter upper bounds of the social preference parameter  $\theta$ .

Another way to reduce the size of the identification region is to assume that senders have objectively correct (rational) expectations. This would imply that  $\mathbf{E}(r_i^{\text{invest}})$  and  $\mathbf{E}(r_i^{\text{keep}})$  both coincide with observed average responder behavior,  $\bar{r}^{\text{invest}}$  and  $\bar{r}^{\text{keep}}$ , and are common for all players. Then, the identification region is a line, connecting all values of  $\beta$  and  $\theta$  which solve

$$\Phi(\beta(\bar{r}^{\text{invest}} - \bar{r}^{\text{keep}}) - \beta w + \theta) = \Pr(\text{invest}). \quad (8)$$

The dashed straight line in Figure 3 represents the estimated identification region obtained under the assumption that beliefs are rational, estimated by replacing  $\bar{r}^{\text{invest}}$  and  $\bar{r}^{\text{keep}}$  with the corresponding sample averages. While the rational expectations assumption effectively reduces the identified region to a single line, this assumption is less realistic since beliefs of senders appear to differ from the observed average behavior of responders (see Figure 2).

Finally, we estimated the parameters of our model using the beliefs stated by each sender. To proceed, we replaced the unknown expectations  $\mathbf{E}(r_i^{\text{keep}})$  and  $\mathbf{E}(r_i^{\text{invest}})$  in (1) and (2) with expectations approximated using the cubic spline interpolation method proposed in Bellemare, Bissonnette, and Kröger (2007).<sup>8</sup> We find that the estimated value of  $\beta$  is 0.117 (standard error = 0.065) and is significant at the 5% level

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<sup>8</sup>Cubic spline interpolation allows to approximate expectations with minimal assumptions concerning the shape of the underlying distributions. Bellemare, Bissonnette, and Kröger (2007) show that the bias when approximating a subjective mean is negligible given the number of probability questions answered by each sender.

against the one-sided alternative that  $\beta > 0$ . This suggests that differences in expectations between investing and not investing have a significant impact on the propensity to invest. We further find that the social preference parameter  $\theta$  is 0.569 (standard error = 0.241) and significant at the 5% level against a two-sided alternative.<sup>9</sup> This suggests that social preferences play a significant role in determining investments in the game. Figure 3 plots this point estimate. We find that it lies within the first two identification regions. Hence, we are unable to reject that the model using subjective expectations data is misspecified.

## 5 Conclusion

In this paper we have applied recent developments in the area of partial identification of econometric models to study sender behavior in the investment game. We have shown how bounds around model parameters can be derived under various levels of assumptions concerning the beliefs of players. We have also shown how these bounds can be used to perform specification tests of the validity of subjective expectations data. Our results provide support for the validity of subjective expectations data: point estimates using belief data fall within these bounds. Our point estimates suggest that expected returns of investment and social preferences both play a significant role in determining investments in the game. Cox (2004) finds similar results for the investment game using a design exploiting multiple treatments. We view the present approach as a useful alternative to make inferences in these games using a single treatment.

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<sup>9</sup>The standard errors are possibly a little conservative as they do not account for noise in the approximated expectations.

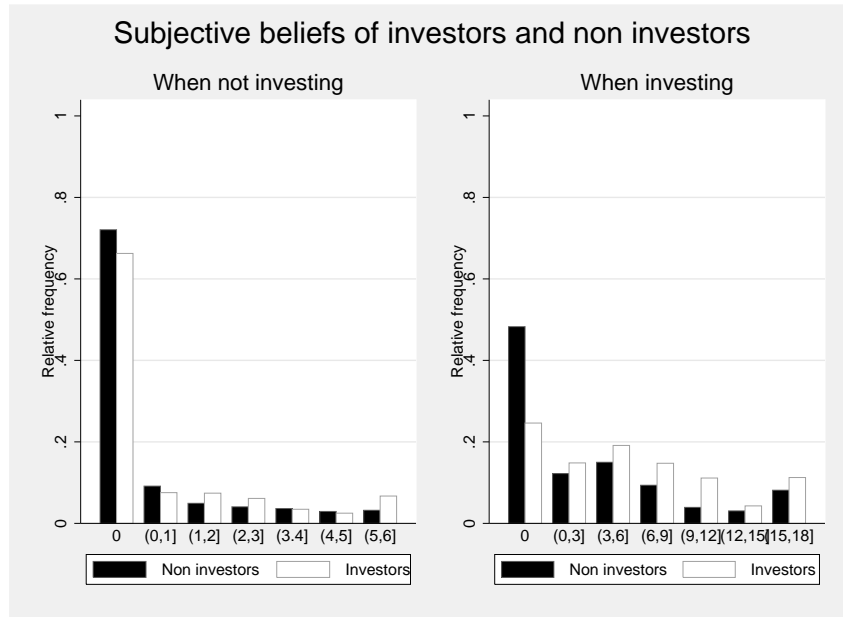


Figure 1: Subjective beliefs about the amount returned separately for investors (light bars,  $N = 24$ ) and non investors (dark bars,  $N = 14$ ) when not investing (left panel) and when investing (right panel).

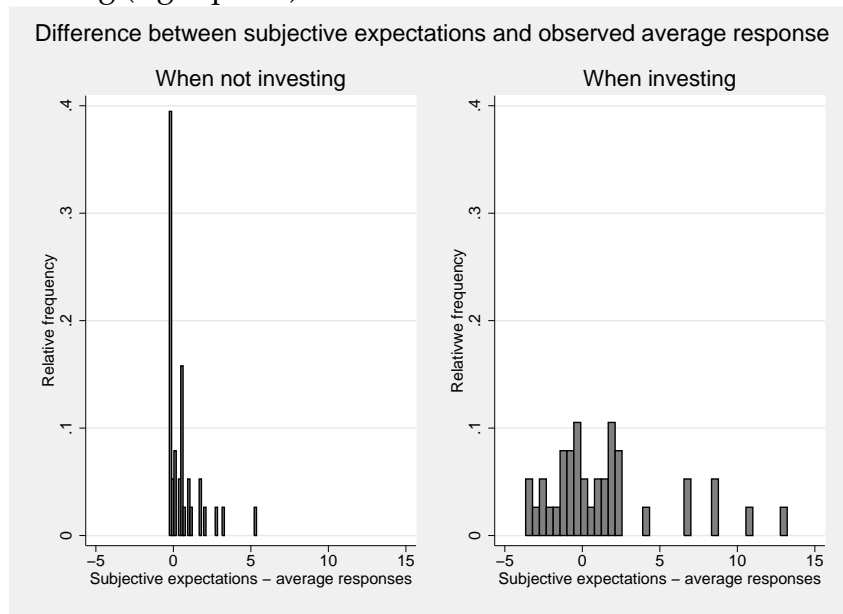


Figure 2: Distribution of the difference between subjective expectations of all first players and observed average response of all second players in the event of investing (top graph,  $N = 38$ ) and in the event of not investing (bottom graph,  $N = 38$ ).

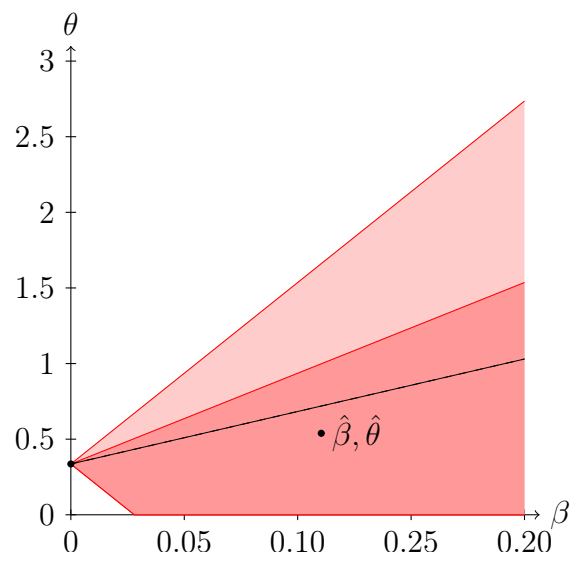


Figure 3: Estimated identification regions without information on subjective expectations (both shaded areas), assuming that  $\mathbf{E}(r_i^{invest}) \geq \mathbf{E}(r_i^{keep})$  (dark shaded area only), and under rational expectations (dashed line). The point  $(\hat{\beta}, \hat{\theta})$  denotes the parameter estimates obtained using the subjective expectations data.

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